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Damage Identification of Welded Structures Using Time Series Models and Exponentially Weighted Moving Average Control Charts

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Abstract

The main aim of this paper is to demonstrate a new approach for the health monitoring of structures to identify the damage at earliest possible stage using the acceleration-time data obtained from the piezoelectric accelerometers. This paper presents a unique combination of time series models to extract the damage sensitive features and exponentially weighted moving average (EWMA) control charts to monitor the variations of the selected features. First, the damage sensitive features are extracted by fitting a time series prediction model called an auto-regressive (AR) model to the acceleration-time data obtained from the undamaged structure. Then the residual errors are calculated which quantify the difference between the actual acceleration-time data and the prediction from the AR model at each time interval is defined as the damage sensitive feature. The variation of these features is monitored using EWMA control charts. The applicability of the proposed damage identification approach is tested with the welded structure like cantilever plate. The damage is introduced to the test structure by cutting a slot in the weld using electrical discharge machining. Three damage levels are considered and named damage level zero, damage level one and damage level two. As the outliers are statistically significant in number and are increasing as the damage level increases, it is concluded from the EWMA control charts that this approach not only identifies the presence of damage but also sensitive to the severity of the damage.

Keywords: Auto regressive model; damage identification; exponentially weighted moving average; acceleration-time data; welded structures.

1. Introduction

Vibration based damage identification is a tool that has received considerable research activity in the field of mechanical, aerospace and civil engineering structures. Most of these structures are welded structures because welding is an economical and efficient method for obtaining a permanent joint. A welded joint offer many advantages like, lighter in weight, less cost, less production time, no stress concentration and provides more strength compared with many other joints [1]. The process of implementing a damage identification strategy for these structures is generally referred to as Structural health monitoring (SHM) [2]. Here damage is defined as changes to the material and/or geometric properties of these systems, including changes to the boundary conditions and system connectivity, which adversely affect the system's performance.

The SHM process involves the observation of a system over time using periodically sampled dynamic response measurements from an array of sensors. Many of these structures continue to be used despite of aging and the associated potential for damage accumulation .Therefore interest in the ability to monitor the structural health and to detect the damage at earliest possible stage is very important for both economical and life safety point of view. Ideally a robust damage detection method will be able to identify the damage at a very early stage, locate the damage with in sensor resolution being used, and provide some estimate of the severity of the damage. Current damage identification methods are either visual or localized experimental methods such as acoustic or ultrasonic methods, magnetic field methods, radiography, eddy-current methods and thermal field methods [3]. All these experimental techniques require that the vicinity of the damage is known a priori and that the portion of the structure being inspected is readily accessible. Subjected to these limitations the need for the additional global vibration based damage identification methods that can be applied to complex structures has lead to the development of methods that examine changes in vibration characteristics of the structures [4, 5 and 6]. Most of the literature show many different methods for extracting damage sensitive features from vibration response measurements. But few of the cited references take a statistical approach for quantifying the observed changes in those features [2]. The extraction of damage sensitive features from these measurements and the statistical analysis of these features are then used to determine the current state of system health. There are other techniques which use the lamb wave parameters to identify the damage [7].

The basic idea of this global damage identification method is that damage will alter the stiffness, mass or energy dissipation properties of a

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system, which in turn alter the measured dynamic response of the system. Therefore all vibration based damage identification methods, namely [8, 9, 10 and 11] depend on experimental data with inherent uncertainties. There are many cases where damage causes a structure to go from a system that can be accurately modeled as a linear system to a system that exhibits a non-linear dynamic response [12]. Common examples of this change in system response are associated with the formation of fatigue cracks that open and close during subsequent dynamic loading and the loss of preload in bolted connections which results in a rattle.

This paper will present the problem of vibration based damage identification method using control chart analysis paradigm, which is one of the most popular method of statistical process control [13, 14]. The applicability of the proposed damage identification approach is tested with the welded structure and the acceleration-time data is collected for the both undamaged and damaged cases .Control charts approach is very efficient and suitable for on line continuous monitoring of the systems [14]. Full automation of the damage identification procedure is necessary for remote i.e., web based monitoring applications.

2. Mathematical Formulation

An AR model is first fitted to the measured acceleration-time histories obtained from the undamaged structure. Residual errors, which quantify the difference between the actual measured time history and the prediction from the AR model at each time interval, are used as the damage-sensitive features. Exponentially weighted moving average (EWMA) control charts are employed to monitor the variation of the selected features. Control limits for the control charts are constructed based on the features obtained from the initial intact structure. The residual errors computed from the new data and the prediction from the AR model are then monitored against the control limits. A statistically significant number of residual errors outside the control limits indicate a system anomaly.

2.1 AR Model

The basic assumption in the use of control charts is the independence of the extracted features. Conventional control charts give false alarms too frequently if the selected features exhibit a high level of correlation over time [15]. Hence it is necessary to remove the correlation in the raw time history before the application of the control charts. As a feature extraction process, an AR model is fitted to the undamaged acceleration time history in order to remove the auto-correlation.

An AR model is essentially an infinite impulse response filter with some additional interpretation placed on it. The notation AR(p) refers to the autoregressive model of order p. The AR(p) model given in [15] is

$$\hat{X}_t = \sum_{i=1}^{p} a_i (X_{t-i}) + \varepsilon_t \qquad (1)$$

where $\hat{X}_t$ is the estimate of the t^{th} time series value, a_1, a_2.......a_p are the parameters or the co-efficients of the AR model, p is the order of the AR model, X_{t-i} previous measured time series values. ε_t is assumed to be an unobservable random error with zero mean and constant variance (white noise). Here white noise is generated using the Mat lab version 7.0 and is presented in later sections.

If $\hat{X}_t$ represents the estimated acceleration-time measurements from the fitted AR model and X_t represents the measured acceleration-time data from the experiment, then the residuals at time 't' is given by

$$e_t = X_t - \hat{X}_t \qquad (2)$$

If the fitted AR model accurately represents the measured signal, the residual should be nearly uncorrelated.

2.1.1 Selection of AR model order 'p'

There are many techniques available for selecting the model order 'p', such as final prediction error (FPE), Akaike's information criteria (AIC) and Bayesian information criteria (BIC). For feature estimation it is not good to select model order p arbitrarily large. Selecting very high order model will results in small estimated white noise varience.

In 1969 Akaike[16] developed FPE criterion to choose the appropriate AR model order to fit to a time series data. By applying FPE criterion select the value of p which will minimizes the FPE. In 1973 Akaike[17] developed a more general applicable criterion for selecting the model order is the information criterion of Akaike known as AIC. But in 1989 Hurvich and Tsai[18] suggested a bias-corrected version of the AIC known as AICC. From AICBIC criteria the order of the AR model is obtained as 4. Therefore AR(4) model is developed from the acceleration-time data of the undamaged structure.

2.1.2.Calculation of AR parameters

The AR parameters are generally calculated by considering the Yule-walker equations[19]. The AR(p) model given in the equation (1) is based on parameters a_i where i = 1, ..., p. There is a direct correspondence between these parameters and the covariance function of the process, and this corrrspondence can be inverted to determine the parameters from the autocorrelation function. This is done by using the following Yule-Walker equations,

$$T_p\, a = \gamma_p \qquad (3)$$

$$\sigma^2 = \gamma(0) - a'\, \gamma_p \qquad (4)$$

where T_p is the covariance matrix $[\gamma(i-j)]_{i,j=1}^{p}$ and $\gamma_p = (\gamma(1) \ldots\ldots \gamma(p))'$

The above equations (the Yule-Walker equations) provide one route to estimate the parameters of an AR(p) model, by replacing the theoretical covariances with estimated values. One way of specificying the estimated covariances is equivalent to calculation using least squares regression of values X_t on the 'p' previous values of the same series.

Once the AR model is fitted to the acceleration- time history obtained from the undamaged structure, $\hat{x}_t$ is the predicted time history from the AR model at time 't'. Then the residual errors(e_t) are calculated using equation(2) and is defined as the damage sensitive features used in this work. The control charts provide statistical frame work to detect the changes in the selected damage sensitive features.

2.2 Statistical Process Control

Control charts may be used in variety of ways, but in many applications it is used for on-line process monitoring. General theory of control charts was first proposed by Dr Walter S. Shewhart[15], and control charts developed according to his principles are often called "Shewhart control charts". Basically control chart is a graphical display with limit lines, called control lines.

The purpose of drawing a control chart is to detect any changes in the process that would be evident by any abnormal points listed on the graph from the data collected. If these points are plotted in "real time", the operator will immediately see that the point is exceeding one of the control limits, and can make an immediate action.

When the structure is in good condition, the damage sensitive features derived from the acceleration-time measurements will have some distribution. These features may change if the structure is damaged. Therefore statistical process control provides a framework for monitoring the features and for identifying new data that are inconsistent with past data. EWMA control charts hitherto not used for the present purpose are proposed to monitor the damage sensitive features derived from the acceleration-time measurements. These control charts are very effective against small process shifts.

2.2.1 EWMA control charts

EWMA control charts are generally considered somewhat more advanced techniques than the Shewhart control charts. EWMA quality control chart offers considerable performance improvement relative to Shewhart quality control charts when the magnitude of the shift in process mean is small. The EWMA control chart was introduced by Robert in 1959[20]. For individual observations (n=1) the EWMA chart is defined as

$$z_i = \lambda x_i + (1-\lambda) z_{i-1} \qquad (5)$$

Where λ is a constant lies between 0 and 1 and the starting value z_0 =CL is the target mean. Recursively substituting $\lambda x_{i-j} + (1-\lambda) z_{i-j-1}$ for z_{i-j}, j=1,2…….i-1 in equation (5), it can be shown that z_i is a weighted average of all past and current observations. Then

$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0 \qquad (6)$$

If the observations x_i are independent random variables with variance σ^2, the variance of z_i is given by the equation,

$$\sigma_{z_i}^2 = \sigma^2 \left(\frac{\lambda}{2-\lambda}\right)[1-(1-\lambda)^{2i}] \qquad (7)$$

Therefore, the EWMA control chart would be constructed by plotting z_i versus the sample number i (or time). The upper control limit (UCL), center line (CL) and lower control limit (LCL) for the EWMA control chart is defined as follows

$$\begin{aligned} UCL &= \bar{x} + L\sigma \sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]} \\ CL &= \bar{x} \\ LCL &= \bar{x} - L\sigma \sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]} \end{aligned} \qquad (8)$$

Where, L and λ are the design parameters of EWMA control chart. Since the EWMA control chart can be viewed as a weighted average of all the past and present observations, the distribution of z_i can be reasonably approximated by a normal distribution as a result of the central limit theorem. Therefore, the EWMA chart is insensitive to the normality assumption of individual observations x_i.

3. Experimental setup

The applicability of the proposed damage identification approach is tested with the test structure (Figure 1) by fixing it to the multi axes electro dynamic vibration shaker. Test structure is made of carbon-steel and the two plates are welded to form a cantilever.

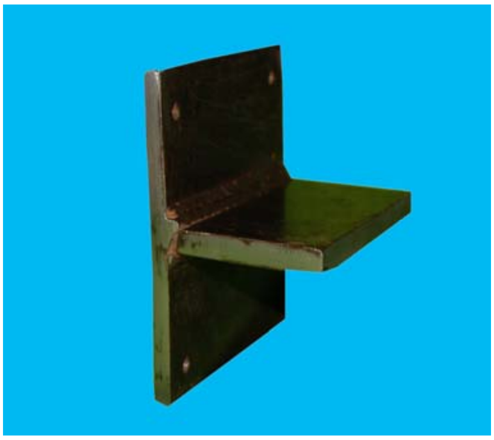
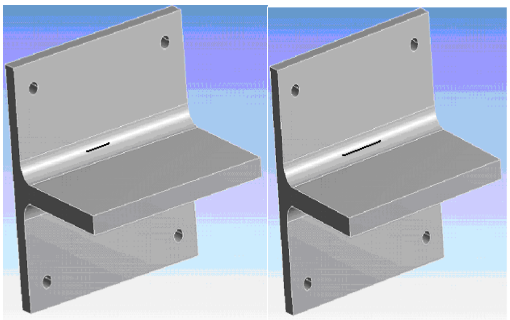


Figure 1. Test structure

The dimensions of the plate which is drilled with four 8 mm through holes at each corner are 150 mm x 150 mm. The centre of the drilled holes is at 18 mm from each corner side of the plate. The dimensions of the other plate which is welded to the above plate, so as to form a cantilever plate is150 mm x 100 mm. The elastic constants of the material considered are Young's modules (E), 200 x 10^9 N/m^2 (200 Gpa), Poisson's ratio (ν), 0.3 and the mass density (ρ), 7850 kg/m^3.

Damage is introduced into the structure by cutting a slot in the weld. This is done by electrical discharge machining (EDM). Two damage levels are investigated by introducing two such slots in different test structures in which 10 mm slot length is considered as damage level one, where as 20 mm slot length is considered as damage level two (Diagrammatically represented in Figure 2). However damage level zero refers to the undamaged condition of the structure. The thickness of the slot is 0.6 mm for all the cases.



a) Damage level one b) Damage level two

Figure 2. Diagrammatic representation of damaged structure

The test structure is fitted to the multi axes electro dynamic vibration shaker with the help of four bolts and nuts as shown in Figure 3. Electro-dynamic vibration shaker used for experimentation consists of drive coil connected rigidly to the moving platform and positioned in the magnetic field when an alternate current flow in this drive coil gives rise to a force by converting an electric current into mechanical force which moves the platform. The vibrator can operate from either sine or random input wave form in the required frequency range.

Figure 3. Experimental setup

Table 1. Specifications of multi axes electro dynamic vibration shaker

Peak sine force	± 400 Kg force
Max. displacement	25 mm(pk-pk)
Max. Velocity 1.2meter/second(nominal)	
Frequency range	1 Hz to 3000 Hz
Size of moving top platform	160 mm diameter
Max. acceleration on vibrator platform	80 g (at no load)
Max. payload capacity	70 Kg's
Moving armature suspension	Rolling strut type
Moving mass of armature	5 kg
Drive power power amplifier	Through a solid state
Cooling method	Air extraction
Test direction	All three mutually perpendicular directions

Table 1 shows the specifications of the multi axes electro dynamic vibration shaker. Vibration characteristics of a structure can be examined in either actively or passively. In this work active investigation is selected for monitoring the vibration characteristics of a structure. The test structure is instrumented with three piezoelectric single axis accelerometers. Out of three accelerometers one is used for actuator (input) and the other two is for response. The response data from the two accelerometers are recorded and was sampled at 150 Hz. Piezoelectric sensors are electromechanical systems that react on compression the sensing elements show almost zero deflection. Due to this reason the piezoelectric sensors are robust, have an extremely high natural frequency and an excellent linearity over a wide amplitude range. Vibration response data from the structure is recorded using data recorder shown in Figure 4.

Figure 4. Response data recorder

Four channel data acquisition system shown in Figure5 is used to convert the analog data into digital form. Total of 1024 acceleration-time measurements are acquired for damage level zero, damage level one and damage level two and are stored in MS-excel file. The plots of this data for different damage levels are shown in Figure 6. Figure 7 shows the white noise generated using Mat Lab 7.0 version.

Figure 5 Four channel data-acquisition system

In general selection of the model either physics-based or data based will depend upon the amount of relevant data available and the level of confidence. When nothing is known a priori, about the structure then approach becomes entirely data based **[**21**]**. The data based models [22, 23, and 24] are relatively simple to fit to the measured response data and the application of one such model is considered in this work for extracting the damage sensitive features. An AR (4) model is fitted to the acceleration-time history obtained from the undamaged structure. Then the residual errors are calculated, which is the difference between the actual subsequent data and the prediction from the AR (4) model. To compare the residual errors obtained from data measured on the undamaged structure with similar quantities obtained from the damaged structure, EWMA control chart is developed using the acceleration-time data obtained from undamaged structure.

The control limits for EWMA control charts are calculated from the residual errors (features) obtained from the undamaged structure. Then the new data (damaged) are monitored against the control limits. A statistically significant number of residual error terms outside the control limit indicating the structure transit from undamaged to damaged state.

4. Results and discussions

In this study the proposed damage identification approach based on time series models and EWMA control charts were tested on a cantilever plate like welded structure. As explained earlier two damage levels are investigated in this work i.e., damage level one and damage level two. Damage level zero configuration was considered "undamaged" and the structure was assumed to be well described by a linear model when subjected to input excitation.

The original data is checked for autocorrelation and shown in Figure8. It can be noted from the Figure8, that the acceleration-time data obtained from the experiment are auto-correlated. Therefore it is necessary to remove the auto-correlation before the application of control charts. To remove the auto-correlation, a linear time prediction model called an AR model is developed from the acceleration-time data obtained from the damage level zero structure. The order of the AR model is calculated as four from the AICBIC criterion for this work. After obtaining the order of AR model, AR parameters are calculated using Yule-walker equations (equations 3 and 4) and the values are tabulated in table 2.

Table 2. AR Parameters

AR parameter	Calculated value from Yule-walker equations
a_1	-1.3644
a_2	0.6061
a_3	-0.6456
a_4	0.4241

Figure 9 presents the acceleration-time data prediction from the AR (4) model for the different damage levels. Then the residual errors are calculated, which is the difference between actual subsequent acceleration-time data and the prediction from AR (4) model. Since it is not possible to estimate the residual error values less than the model order, a total of 1020 residual errors are calculated from 5 to1024 in each case of damage level and it is presented in Figure10. Then EWMA quality control charts are plotted using residual errors as data. EWMA control charts are plotted for individual observation (n=1) using 1020 residual errors (features) as data. The design parameters of the EWMA control chart L and λ are selected as 2.7 and 0.2 respectively for present case as per [15].

Figure11 shows the EWMA control charts for damage level zero, damage level one and damage level two. This shows that there are statistically significant number of data points are outside the control limits for the two damage levels considered. This is a clear indication of presence of damage in the structure. However it is seen that the number of outliers are more for damage level two than that of the damage level one, which is clear indication that this approach is sensitive to the severity of damage. It has been observed from Figure 11 (b) for damage level one 66 residual error points are falling outside the control limits, and for damage level two from Figure 11(c), 105 residual error points are falling outside the control limits. Therefore this damage identification approach not only identifies the damage but also sensitive to the severity of the damage.

5. Conclusions

In this paper a statistical damage classification technique for vibration based damage identification problem in an unsupervised learning mode is studied. A unique combination of linear prediction model called an AR model to extract the damage sensitive features and EWMA control charts to monitor the variation of the selected features is presented. An experiment on welded structure was conducted using multi axes electro dynamic vibration shaker to study the applicability of statistical damage classification technique.

The experimental data obtained from the piezoelectric sensors are auto correlated. To remove the auto correlation an AR (4) model is developed from the acceleration-time data obtained from the damage level zero. Then the residual errors are calculated which quantify the difference between the actual measured time history and the prediction from the AR (4) model at each time interval, are used as the damage-sensitive features. Here two damage levels are considered i.e., damage level one and damage level two. The residual errors are nearly uncorrelated therefore the residual errors are taken as data for plotting the EWMA control charts. It is observed from the EWMA control charts that there are statistically significant number outliers. This suggests the presence of damage in the structure. These outliers are increasing as the damage level increases. This shows that this approach is sensitive to the severity of the damage. Therefore the approach presented in this paper is effective in identifying damage and also identifying its severity in the considered welded structure. Once the presence of damage is conformed in any machine structure it can be inspected thoroughly and can be put back into further service.

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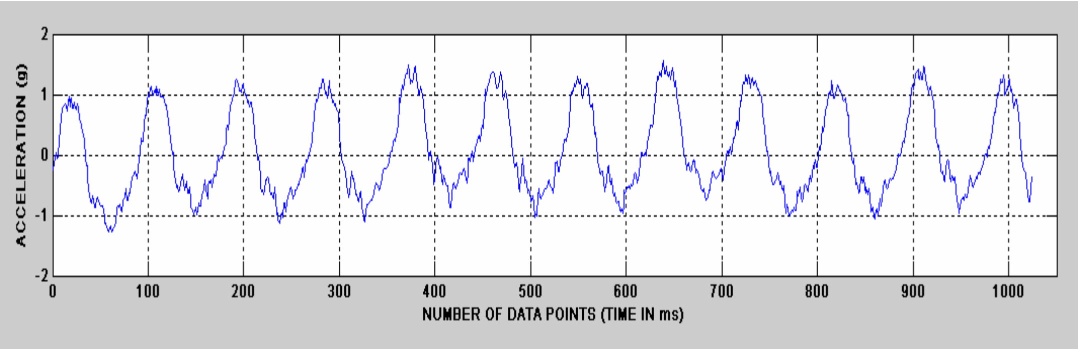
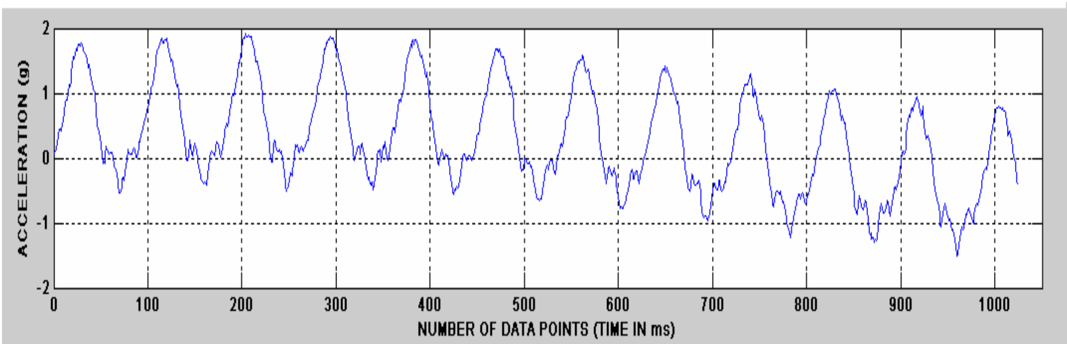
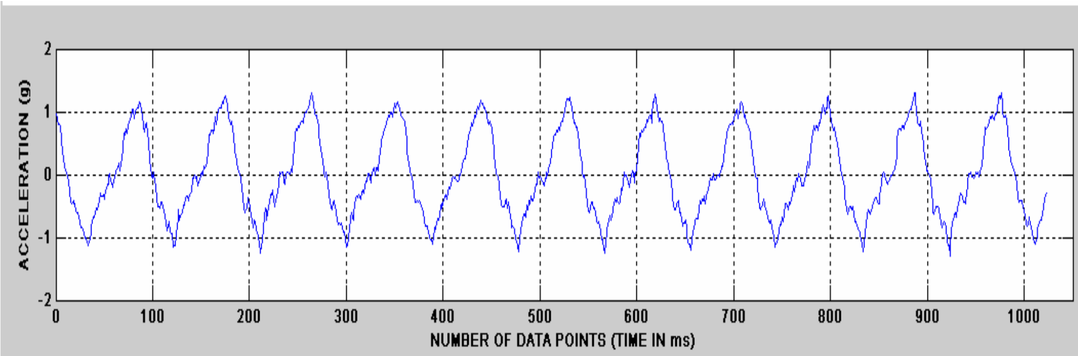


6(a) Damage level zero



6(b) Damage level one



6(c) Damage level two

Figure 6. Acceleration-time data from the experiment for different damage levels

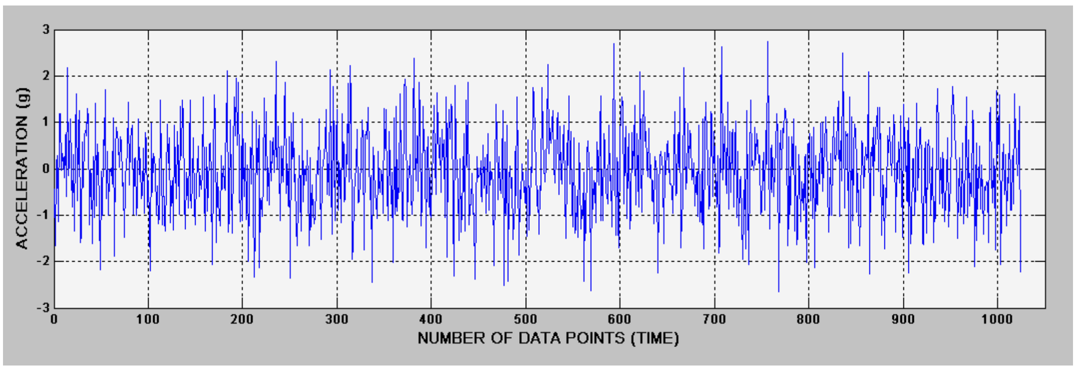
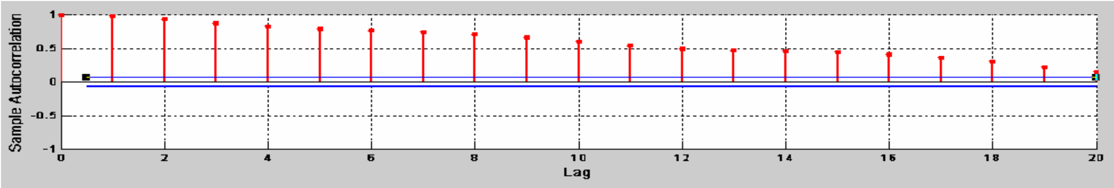
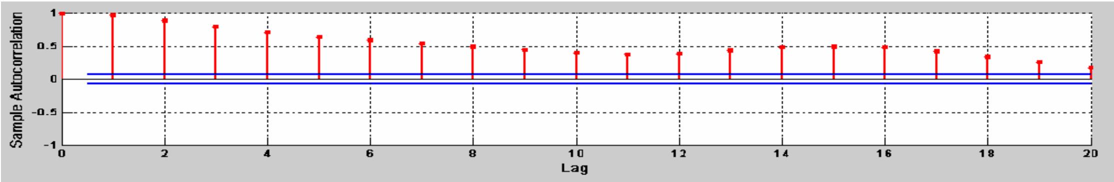


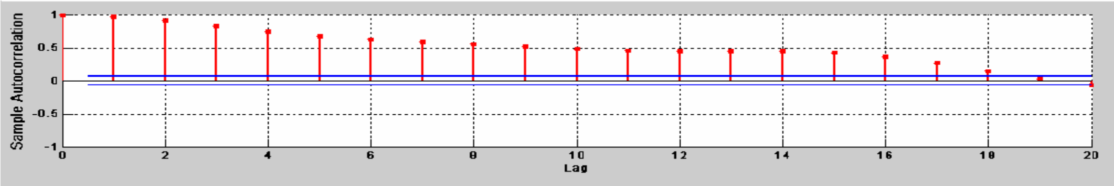
Figure 7. White noise generation with zero mean and constant variance



8(a) Damage level zero

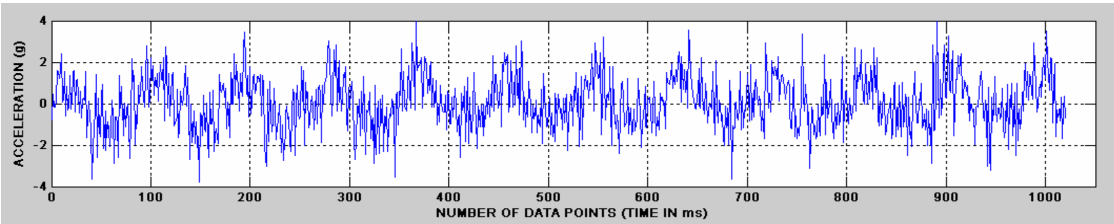


8(b) Damage level one

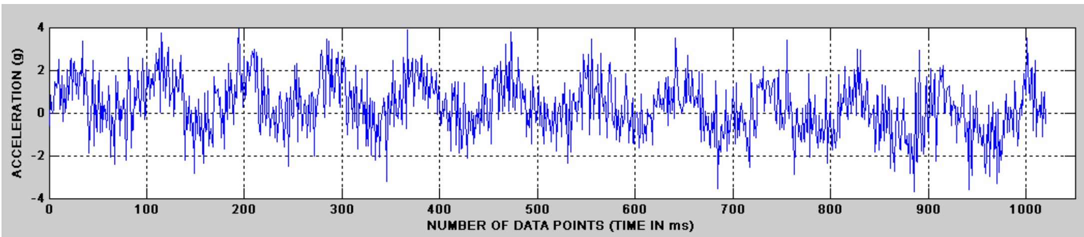


8(c) Damage level two

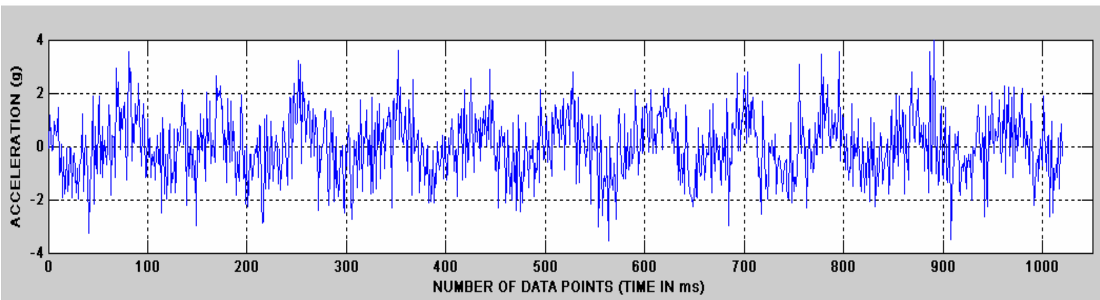
Figure 8. Auto-correlation for the experimental data obtained from different damage levels.



9(a) Damage level zero

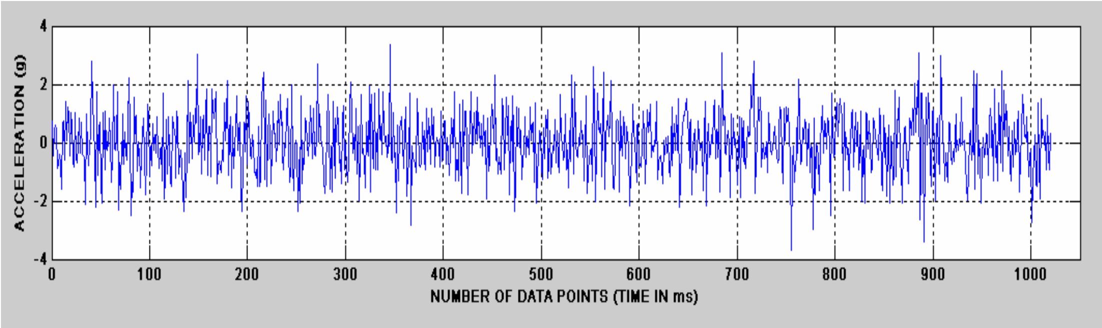


9(b) Damage level one

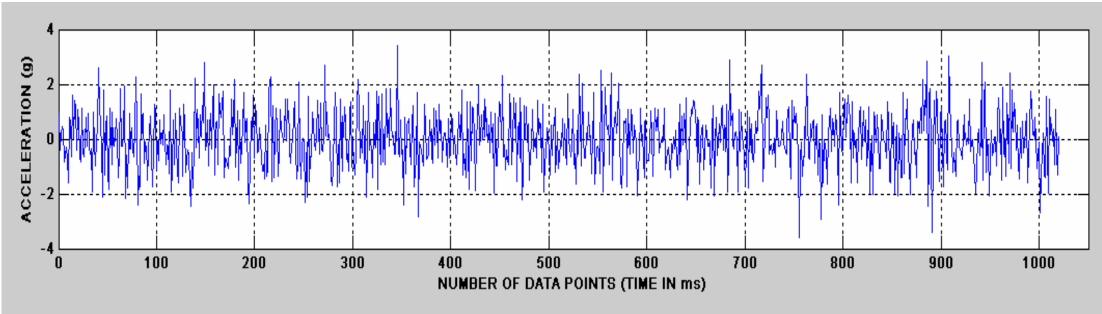


9(c) Damage level two

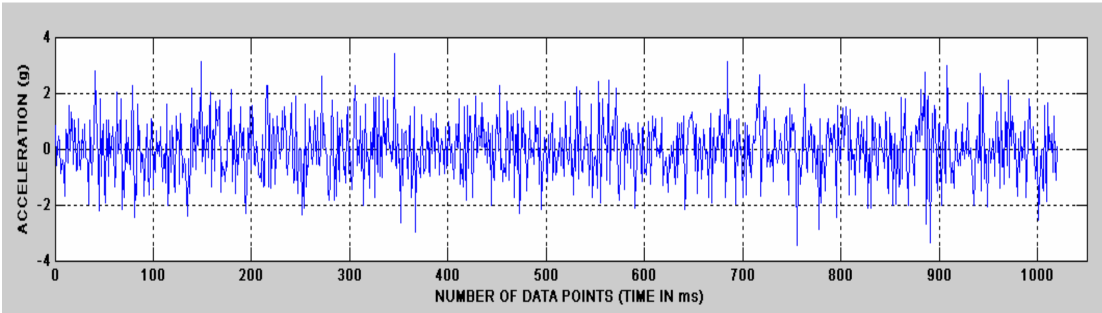
Figure 9. Acceleration-time data prediction from AR (4) model for different damage levels



10(a) Damage level zero

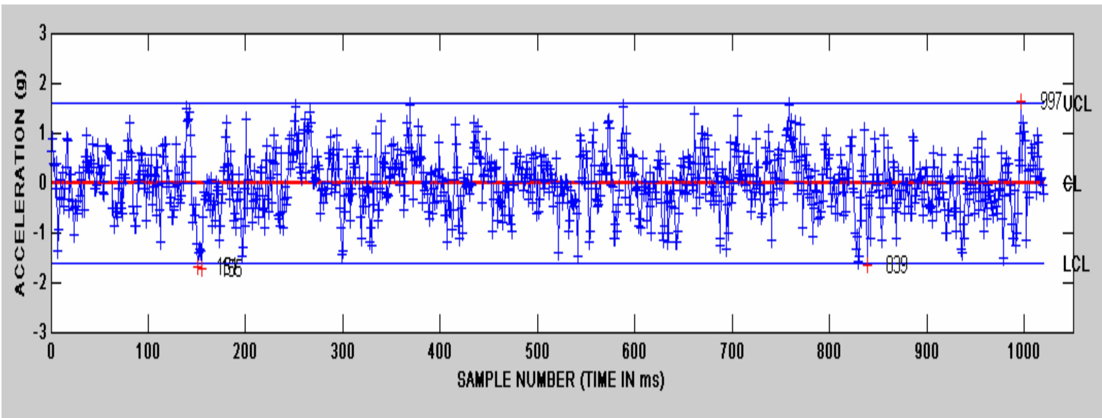


10(b) Damage level one

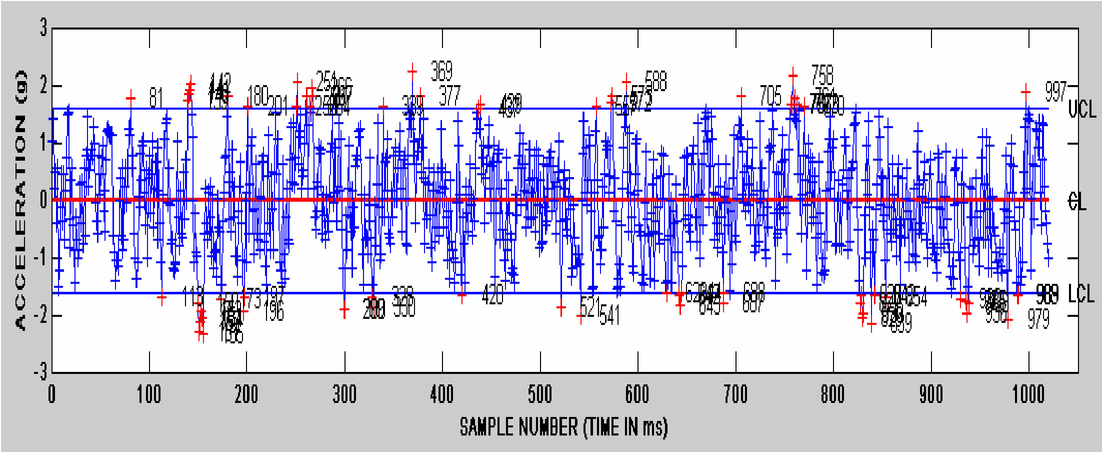


10(c) Damage level two

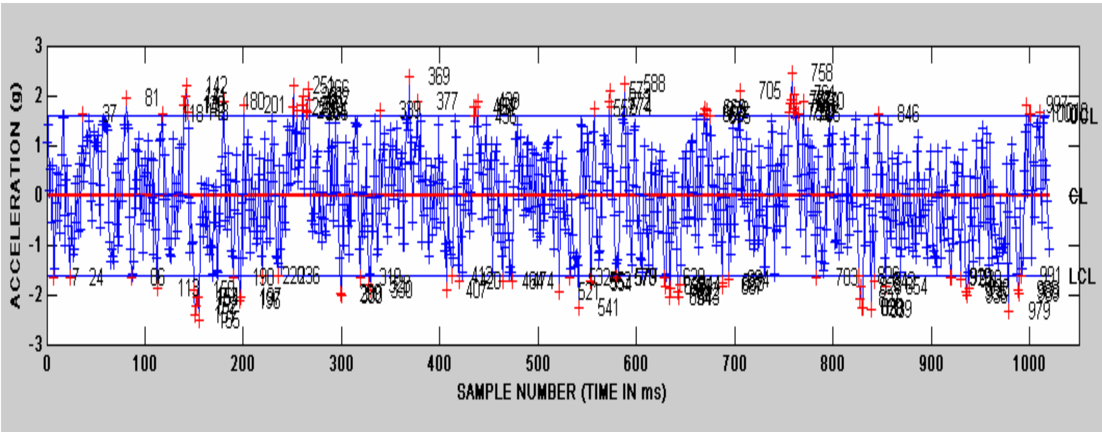
Figure 10. Residual errors (damage sensitive features) for different damage levels



11(a) Damage level zero



11(b) Damage level one



11(c) Damage level two

Figure 11. EWMA control charts (n=1) for residual errors obtained from different damage levels