

Real Time Compensation of Machining Errors for Machine Tools NC Based on Systematic Dispersion

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Abstract

Manufacturing tolerancing is intended to determine the intermediate geometrical and dimensional states of the part during its manufacturing process. These manufacturing dimensions also serve to satisfy not only the functional requirements given in the definition drawing, but also the manufacturing constraints, for example geometrical defects of the machine, vibration and the wear of the cutting tool. In this paper, an experimental study on the influence of the wear of the cutting tool (systematic dispersions) is explored. This study was carried out on three stages. The first stage allows machining without elimination of dispersions (random, systematic) so the tolerances of manufacture according to total dispersions can be identified. In the second stage, the results of the first stage are filtered in such way to obtain the tolerances according to random dispersions. Finally, from the two previous stages, the systematic dispersions are generated. The objective of this study is to model by the least squares method the error of manufacture based on systematic dispersion. Finally, an approach of optimization of the manufacturing tolerances was developed for machining on a CNC machine tool

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1. Introduction

Manufacturing tolerancing is intended to determine the intermediate geometrical and dimensional states of the part during its manufacturing process. These manufacturing dimensions also serve to satisfy not only the functional requirements given in the definition drawing, but also the manufacturing constraints, for example geometrical defects of the machine, vibration and the wear of the cutting tool...

Many research works were treated the tolerancing problem with different approaches, Rong and Bai [1] analyzed a dependent relationship of operational dimensions to estimate machining errors in terms of linear and angular dimensions of a workpiece. Cai et al. [2] proposed a method to conduct a robust fixture design to minimize workpiece positional errors as a result of workpiece surface and fixture setup errors. Djurdjanovic and Ni [3] developed procedures for determining the influence of errors in fixtures, locating datum features and measurement datum features on dimensional errors in machining. These studies were conducted when a static case was assumed.

Kim and Kim [4] have developed a volumetric error model based on 4x4 homogenous transformations for generalized

geometric error. Eman and Wu [5] have developed error model accounts for error due to inaccuracies in the geometry and mutual relationships of the machine structural elements as well as error resulting from the relative motion between these elements. Kakino et al. [6] have measured positioning errors of multi-axis machine tools in a volumetric sense by Double Ball Bar (DBB) device. Takeuchi and Watanabe [7] have shown five-axis control collision free tool path and post processing for NC-data.

In the work of [8], the authors present an experimental semi study of the vibratory behavior of the cutting tool during the operation of slide-lathing, is the object to show that it is possible to consider the roughness average of the part machined starting from displacement resulting from the nozzle of the tool. In the work of [9] a study was presented on the influence of the position of the cutting tool on dynamic behavior in milling of thin walls, and in work of [10,11,12], authors thus illustrate the influence of the trajectory of the cutting tool on the surface quality tolerances of manufacture for machining on the machine tool has numerical control.

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2. Sources of errors

Before we look at how the task of error compensation can be achieved, we need to clearly understand what we mean by accuracy and error. Accuracy could be defined as the degree of agreement or conformance of a finished part with the required dimensional and geometrical accuracy [13]. Error, on the other hand, can be understood as any deviation in the position of the cutting edge from the theoretically required value to produce a workpiece of the specified tolerance. The extent of error in a machine gives a measure of its accuracy; that is the maximum translation error between any two points in the work volume of the machine. This of course depends on the resolution of the system. Positioning can never be more accurate than this as there will be no further feedback to improve the positioning within this range. However, more important than system resolution, are the errors that occur between the measurement point and the feedback point [14]. The best way to keep track of the errors is to formulate an error budget. An error budget allocates resources among the different components of a machine. It is a system analysis tool used for the prediction and control of the total error of a system. An error budget basically addresses two fundamental issues. One involves obtaining the influence of different sources of error (the individual members of the kinematic chain of the machine tool) on the accuracy of the machine. The other involves taking a set of specifications and determining the permissible level of each source so that some criterion like cost etc., is optimised [15]. Errors can be classified into two categories namely quasi-static errors and dynamic errors. Quasi-static errors are those between the tool and the workpiece that are slowly varying with time and related to the structure of the machine tool itself. These sources include the geometric/kinematic errors, errors due to dead weight of the machine's components and those due to thermally induced strains in the machine tool structure. Dynamic errors on the other hand are caused by sources such as spindle error motion, vibrations of the machine structure, controller errors etc. These are more dependent on the particular operating conditions of the machine. Quasistatic errors account for about 70 percent of the total error of the machine tool and as such, are a major focus of error compensation research. Once the individual error components have been identified, the next step in the problem of error budgeting is to determine the optimal level of these errors so that the cost factor is minimised [15].

In general, a machining centre consists of a bed, column, spindle and its slide and the various linear and/or rotary axes. Each of these elements contributes to the total error of the system that is represented by the error budget. Errors can broadly be classified as:

- a) Geometric errors of machine components and structures
- b) Kinematic errors
- c) Errors induced by thermal distortions
- d) Errors caused by cutting forces including
 - (i) by gravity loads
 - (ii) by accelerating axes, and
 - (iii) by the cutting action itself
- e) Material instability errors

- f) Machine assembly-induced errors
- g) Instrumentation errors
- h) Tool wear
- i) Fixturing errors and
- j) Other sources of errors like servo errors of the machine (following errors and interpolation algorithmic errors) [16,17,18].

2.1. Geometric and kinematic errors

Geometric errors are those errors that are extant in a machine on account of its basic design, the inaccuracies built-in during assembly and as a result of the components used on the machine.

As such, they form one of the biggest sources of inaccuracy. These errors are concerned with the quasi-static accuracy of surfaces moving relative to one another. Geometric errors can be smooth and continuous or they could exhibit hysteresis or random behaviour. These errors are affected by factors like surface straightness, surface roughness, bearing pre-loads etc. Geometric errors have various components like linear displacement error (positioning accuracy), straightness and flatness of movement of the axis, spindle inclination angle, squareness error, backlash error etc [17]. Kinematic errors are concerned with the relative motion errors of several moving machine components that need to move in accordance with precise functional requirements. These errors are particularly significant during the combined motion of different axes as in the case of gear hobbing or profile machining where co-ordination of rotary with respect to linear axes or linear with respect to linear axes is of prime importance. Such errors occur during the execution of linear, circular or other types of interpolation algorithms and are more pronounced during actual machining.

2.2. Thermal errors

Another principal cause for inaccurate workpieces is error due to improper tool positioning on account of thermal deformation. It is well understood that errors due to thermal factors account for 40–70% of the total dimensional and shape errors of a workpiece in precision engineering [19]. Six sources of thermal influence are identified: (i) heat from the cutting process, (ii) heat generated by the machine, (iii) heating or cooling provided by the cooling systems, (iv) heating or cooling influence of the room, (v) the effect of people and (vi) thermal memory from any previous environment [19]. Critical among these sources is heat generated by the machine. Continuous running of the machine causes heat to be generated at the moving elements as a result of frictional resistance, at the motors, in pumps etc [20]. This heat causes relative expansion of the various elements of the machine tool leading to inaccurate positioning of the cutting tool tip.

Consequently errors due to spindle growth, thermal expansion of the ballscrews and thermal distortion of the column are generated at the tool tip. As heat generation at contact points is unavoidable, this source of error is one of the most difficult to eliminate completely. In the manufacture of precision components error due to thermal deformation of the machine elements plays a vital role in limiting the accuracy of the part produced.

2.3. Cutting-force induced errors

The dynamic stiffness of all the components of the machine tool (namely the bed, column, etc.) that are within the force-flux flow of the machine is responsible for errors caused as a result of the cutting action. This is one of the major sources of error in metal-cutting machines as the force involved in the cutting action is considerable. As a result of the forces, the position of the tool tip with respect to the workpiece varies on account of the distortion of the various elements of the machine. Depending on the stiffness of the structure under the particular cutting conditions, the accuracy of the machine tool would vary. Thus, for a machine with a given stiffness a heavy cut would generally produce more inaccurate components than a light cut.

2.4. Other errors

Other errors like tool wear and fixturing errors add to the overall inaccuracy of the machined component. Errors in fixturing are caused by fixture set-up and geometric inaccuracies of the locating elements and by fixture flexure. In cases where the workpiece is restrained by a small area of contact with the fixture, the errors due to deformation or lift-off of the workpiece could cause significant errors. Workpiece displacement is dependent on several factors like position of the fixturing elements, clamping sequence, clamping intensity, type of contact surface etc. Thus workpiece displacement could be a significant source of machine error.

3. Systematic dispersion

Systematic dispersion is due primarily to the wear of the cutting tool between the realization of the first part and the last part of a given series (Figure1).

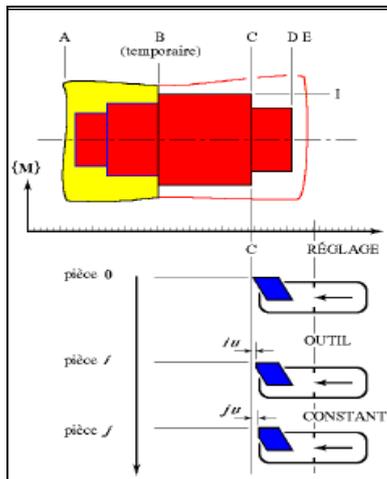


Figure 1: Wear of tool [22]

The wear of the tool leads us to make an experimental study which makes it possible to show the influence of systematic dispersion on the manufacturing tolerances.

4. Procedure of the tests

It is difficult, if not impossible; to obtain manufacturing tolerances while being limited only to systematic dispersions. For this reason, it is necessary to take into account all dispersions. In order to achieve this goal, there are three stages:

1st stage:

The machining of the parts is done without the elimination of dispersions (random, systematic) so that one finds the manufacturing tolerances according to total dispersions.

2nd stage:

The results of the first stage were filtered in such way that one only finds the tolerances of manufacture according to random dispersions.

3rd stage:

From the two previous stages, we compute the systematic dispersions.

5. Conditions of the tests

We have machined 40 parts, C35 matter, on lathe with numerical control using a facing tool with standard brought back pastille “J11ER”

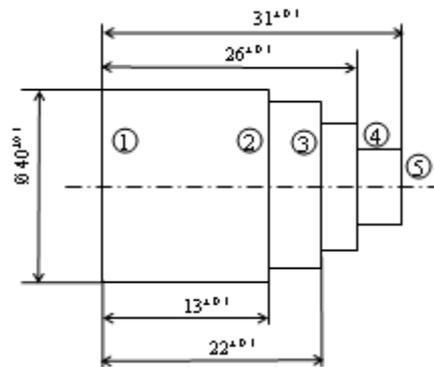


Figure 2: Design drawing of the workpiece.

We place the test in the following conditions:

- The wear of the tool is a linear function;
- Thermal balance is reached;
- Geometrical dispersion is immersed in random dispersion;
- The machining of surface 1 is carried out under phase with the same tool;

6. Stages of manufacture

After having presented the procedure, let us detail the three stages of the study:

6.1. First stage

Starting from a crude, we carried out 5 surfaces on a lathe with numerical control, (Figure 3), by respecting the following parameters of the cut:

- Cutting speed: $C_s = 80$ m/mn;
- Speed in advance: $F = 0.05$ mm/tr;
- machining without lubrication;
- Depth of cut = 2 mm;

A program of machining was developed under a language FANUC (Figure3), for the realization of these tests

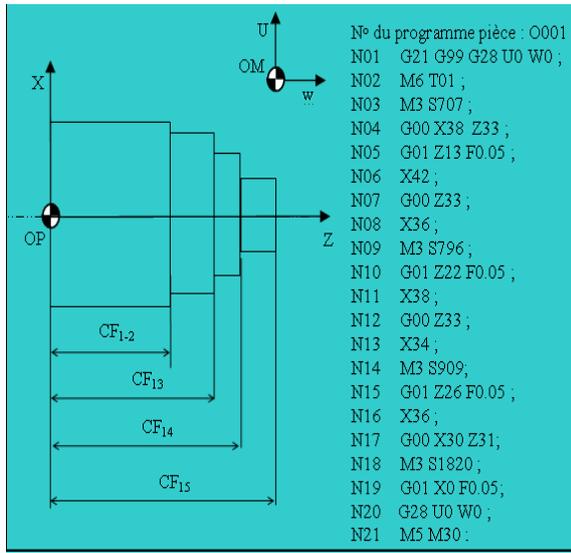


Figure 3: Sketch of turning phase

On each part of the series, we measures dimensions d12, d13, d14, d15 and we calculates d23.

From the equations (1), (2) and (3), we gives the statistical results (the average \bar{X} , the standard deviations σ_{ij} and ΔCF_{ij}) illustrated in Table 1

Table 1: Statistical Results of the 1st stage

	X (mm)	σ (mm)	ΔCF_{ij} (mm)
d12	13.0052	3.489 10 ⁻²	2.093 10 ⁻¹
d13	22.0074	2.860 10 ⁻²	1.716 10 ⁻¹
d14	26.0076	3.199 10 ⁻²	1.917 10 ⁻¹
d15	31.0141	2.685 10 ⁻²	1.611 10 ⁻¹
d23	9.0020	3.369 10 ⁻²	2.021 10 ⁻¹

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N n_i x_i \tag{1}$$

$$v(x) = \sum_{i=1}^N \frac{n_i}{N} (x_i - \bar{x})^2 \tag{2}$$

$$\sigma = \sqrt{v(x)} \tag{3}$$

The system (5), is deduced by the equation (4)

$$\Delta CF_{ij} = \Delta l_i + \Delta l_j \tag{4}$$

$$\begin{cases} \Delta CF_{12} = \Delta l_1 + \Delta l_2 \\ \Delta CF_{13} = \Delta l_1 + \Delta l_3 \\ \Delta CF_{14} = \Delta l_1 + \Delta l_4 \\ \Delta CF_{25} = \Delta l_2 + \Delta l_5 \\ \Delta CF_{23} = \Delta l_2 + \Delta l_3 \end{cases} \tag{5}$$

The resolution of the system (5) leads to the solutions (6):

$$\begin{cases} \Delta l_1 = 8.949 \quad 10^{-2} \text{ mm} \\ \Delta l_2 = 1.200 \quad 10^{-1} \text{ mm} \\ \Delta l_3 = 8.221 \quad 10^{-2} \text{ mm} \\ \Delta l_4 = 1.029 \quad 10^{-1} \text{ mm} \\ \Delta l_5 = 7.199 \quad 10^{-2} \text{ mm} \end{cases} \tag{6}$$

The results (6) represent total dispersions.

6.2. Second stage

In this stage, formula (7) was used to filter dimensions of the first stage.

$$da_{ij} = dt_{ij} - \frac{\Delta CFs_{ij}}{N} i \tag{7}$$

ΔCFs_{ij} : The variation of the dimensions manufactured with systematic dispersion;

da_{ij} : Filtered dimensions;

dt_{ij} : Dimensions according to total dispersion;

i : Number of test;

To calculate Δs_{ij} , we trace for each dij the graphs (dij (N)) and average lines, Figure (4-8).

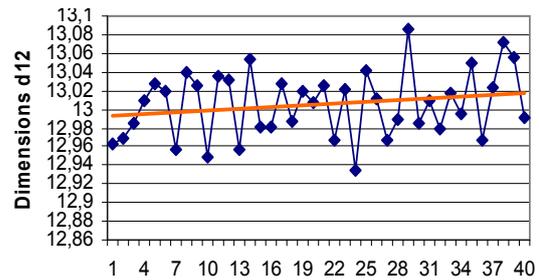


Figure 4: Graph of dimensions d12

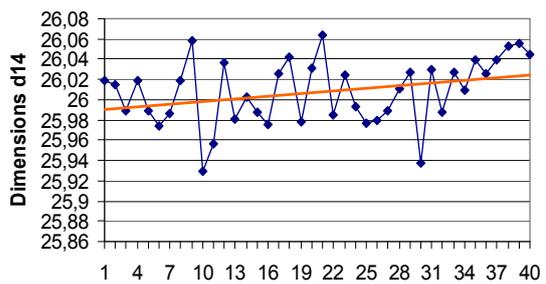


Figure 5: Graph of dimensions d14

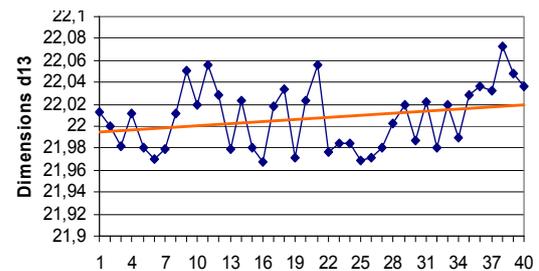


Figure 6: Graph of dimensions d13

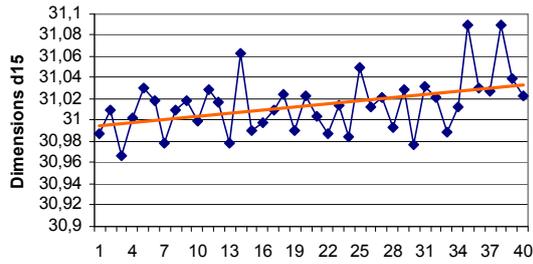


Figure 7: Graph of dimensions d15

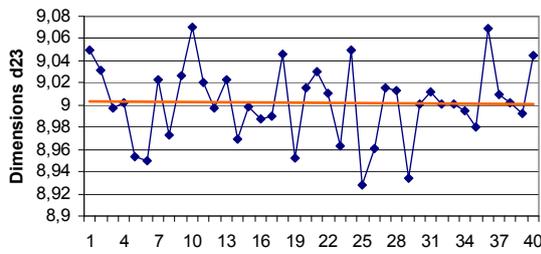


Figure 8: Graph of dimensions d23

The variation of manufacture, expression (8), is given according to the average line tangent a_{ij} and of the tests number N.

$$\Delta CFs_{ij} = a_{ij} \cdot N \quad (8)$$

The resolution of the system (8) gives the results of ΔCFs_{ij} , represented by (9).

$$\begin{cases} \Delta CFs_{12} = 2.656 \cdot 10^{-2} \text{ mm} \\ \Delta CFs_{13} = 2.508 \cdot 10^{-2} \text{ mm} \\ \Delta CFs_{14} = 3.428 \cdot 10^{-2} \text{ mm} \\ \Delta CFs_{15} = 3.816 \cdot 10^{-2} \text{ mm} \\ \Delta CFs_{23} = 0.169 \cdot 10^{-2} \text{ mm} \end{cases} \quad (9)$$

From the equation (7) and the system (9), calculation gives new dimensions (da_{ij}).

Table 2 represents the statistical results calculated starting from the equations (1), (2) and (3)

Table 2: Statistical results of the 2nd stage.

	X (mm)	δ (mm)	ΔCFa_{ij} (mm)
da12	12.992	3.403 10-2	2.041 10-1
da13	21.995	2.767 10-2	1.661 10-1
da14	25.991	3.042 10-2	1.825 10-1
da15	30.995	2.448 10-2	1.469 10-1
da23	9.003	3.369 10-2	2.021 10-1

According to the equation (10), there is the system of expressions (11)

$$\Delta CFa_{ij} = \Delta la_i + \Delta la_j \quad (10)$$

$$\begin{cases} \Delta CFa_{12} = \Delta la_1 + \Delta la_2 \\ \Delta CFa_{13} = \Delta la_1 + \Delta la_3 \\ \Delta CFa_{14} = \Delta la_1 + \Delta la_4 \\ \Delta CFa_{25} = \Delta la_2 + \Delta la_5 \\ \Delta CFa_{23} = \Delta la_2 + \Delta la_3 \end{cases} \quad (11)$$

The resolution of the system (11) leads to the solutions (12):

$$\begin{cases} \Delta a_1 = 8.379 \cdot 10^{-2} \text{ mm} \\ \Delta a_2 = 1.199 \cdot 10^{-1} \text{ mm} \\ \Delta a_3 = 8.200 \cdot 10^{-2} \text{ mm} \\ \Delta a_4 = 9.824 \cdot 10^{-2} \text{ mm} \\ \Delta a_5 = 6.279 \cdot 10^{-2} \text{ mm} \end{cases} \quad (12)$$

The result of system (12) represents random dispersions.

6.3. Third Stage

In this stage we trace the graphs of the dimensions filtered (da_{ij}) according to many tests (N).

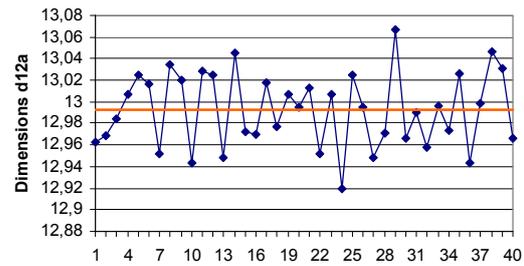


Figure 9: Graph of dimensions da12

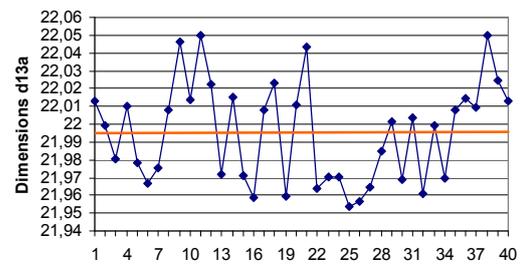


Figure 10: Graph of dimensions da13

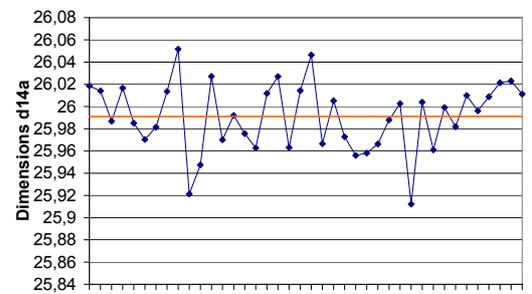


Figure 11: Graph of dimensions da14

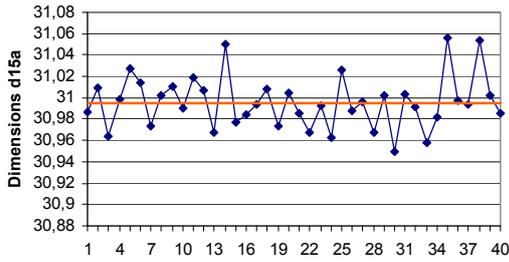


Figure 12: Graph of dimensions da15

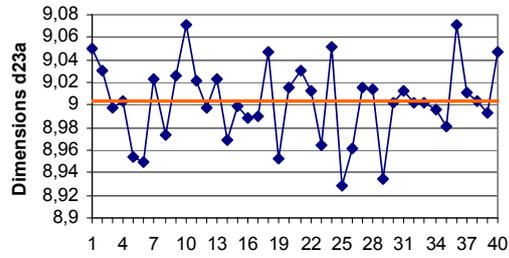


Figure 13: Graph of dimensions da23

According to figures' (9), (10), (11), (12), (13), we notice that the tangents are equal to zero. Therefore filtering is made completely. The replacement of the equation (13) in the system (14), leads to the system (15).

$$\Delta CF_{s_{ij}} = \Delta s_i + \Delta s_j \tag{13}$$

$$\begin{cases} \Delta CF_{s_{12}} = 2.656 \cdot 10^{-2} \text{ mm} \\ \Delta CF_{s_{13}} = 2.508 \cdot 10^{-2} \text{ mm} \\ \Delta CF_{s_{14}} = 3.428 \cdot 10^{-2} \text{ mm} \\ \Delta CF_{s_{15}} = 3.816 \cdot 10^{-2} \text{ mm} \\ \Delta CF_{s_{23}} = 0.169 \cdot 10^{-2} \text{ mm} \end{cases} \tag{14}$$

$$\begin{cases} \Delta s_1 = 2.497 \cdot 10^{-2} \text{ mm} \\ \Delta s_2 = 0.158 \cdot 10^{-2} \text{ mm} \\ \Delta s_3 = 0.00105 \cdot 10^{-2} \text{ mm} \\ \Delta s_4 = 0.931 \cdot 10^{-2} \text{ mm} \\ \Delta s_5 = 1.319 \cdot 10^{-2} \text{ mm} \end{cases} \tag{15}$$

The results of system (15) represent systematic dispersion.

7. Interpretation

Table 3 presents a recapitulation of the results of the dispersions calculated in the three stages.

Table 3: Results of calculated dispersions.

	Total dispersion (mm)	Random dispersion (mm)	Systematic dispersion (mm)
Surface 1	$0.895 \cdot 10^{-1}$	$0.837 \cdot 10^{-1}$	$2.497 \cdot 10^{-2}$
Surface 2	$1.200 \cdot 10^{-1}$	$1.199 \cdot 10^{-1}$	$0.158 \cdot 10^{-2}$
Surface 3	$0.822 \cdot 10^{-1}$	$0.819 \cdot 10^{-1}$	$0.105 \cdot 10^{-3}$
Surface 4	$1.029 \cdot 10^{-1}$	$0.982 \cdot 10^{-1}$	$0.931 \cdot 10^{-2}$
Surface 5	$0.719 \cdot 10^{-1}$	$0.627 \cdot 10^{-1}$	$1.319 \cdot 10^{-2}$
Summon	$4.667 \cdot 10^{-1}$	$4.462 \cdot 10^{-1}$	$0.490 \cdot 10^{-1}$

In figure 17 the graph of total dispersion is almost confused with the graph of random dispersion. The influence of systematic dispersions on the tolerances of manufacture is minimal Compared to random dispersions.

The greatest value of systematic dispersions (Figure 14) is on surface 1 used like reference surfaces, due to the influence of the machined surfaces 2, 3, 4, 5, and the grinding problem of the tool.

On the other hand the greatest value of random dispersions (Figure 15) is on the level of surface 2, then surface 4. The order of surfaces is not imperative; it has a relationship to the setting in position of the part in the chuck, the quality of tightening (manual or pneumatic) or the stop materializing the fifth point of isostatism. The smallest value is at the level of surface 3, since the latter is between two machined surfaces characterized by a small machining length.

The values of systematic dispersions are very small relative with total dispersion. The sum of the values of systematic dispersions is about 10% of the sum of total dispersions. Therefore random dispersion accounts for 90% of total dispersion.

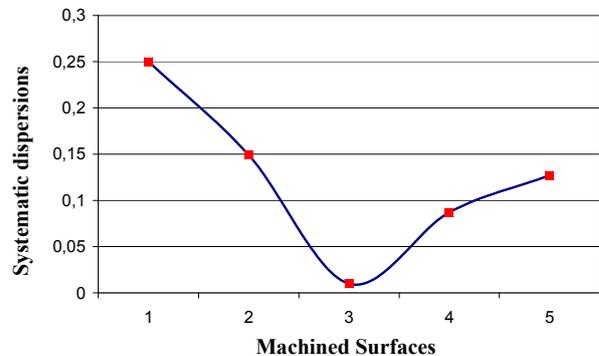


Figure 14: Systematic dispersions according to the machined surfaces.

Figure 15: Random dispersions according to the machined surfaces.

Figure 16 represents the evolution of total dispersions according to the machined surfaces.

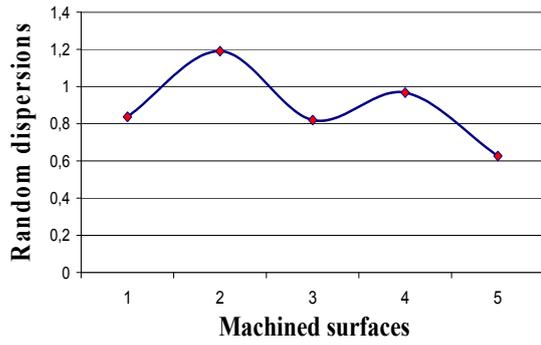


Figure 16: Total dispersions according to the machined surfaces.

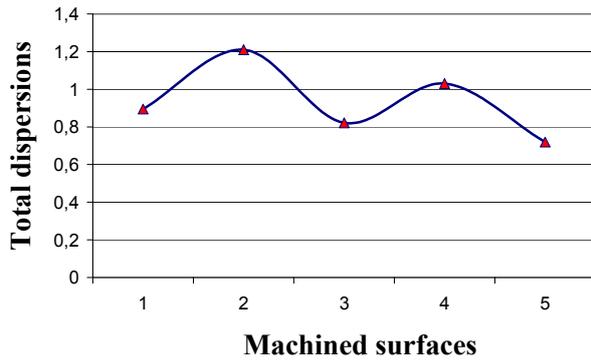


Figure 17 illustrates a comparison between total dispersions and random dispersions.

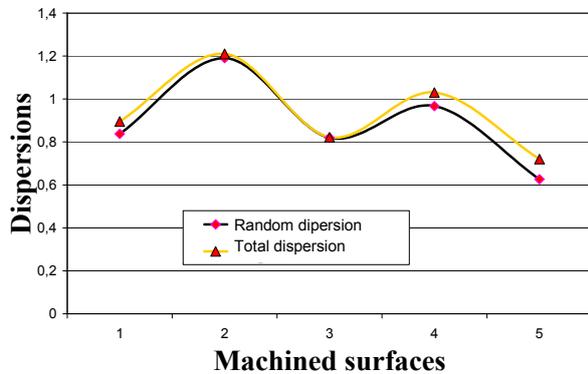


Figure 17: Comparison enters total and random dispersions.

Figure 18 illustrates the percentage between the sum of the values of systematic dispersions and the sum of the values of total dispersions.

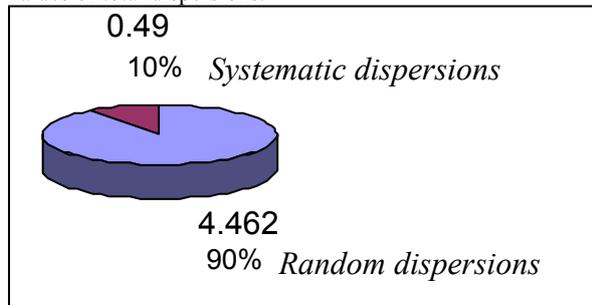


Figure 18: Statistical representation by sector between systematic dispersion and random dispersion.

8. Errors modeling

In this part, a modeling of the errors of dispersions was worked out by the least squares method to develop two models of correction of the tolerances. In the first model, equation 16, we can calculate tolerances of manufacture due to systematic dispersion according to the machined length. In the second model, equation 17, we can calculate tolerances of manufacture according to the machined length

$$IT = 6,5 \cdot 10^{-4} \sqrt[3]{D} + 1,3 \cdot 10^{-5} \cdot D + 1,2 \cdot 10^{-5} \quad (16)$$

$$IT = 0,05 \sqrt[3]{D} + 10^{-3} \cdot D + 10^{-3} \quad (17)$$

After the integration of tolerancing models in the numerical command control program, the new statistical results are given by the Table 4 .

According to the results, the variations of manufacture (tolerance of manufacture) were decrease by 55 % .

Table 4: Statistical Results.

	X (mm)	δ (mm)	ΔCF_{ij} (mm)
d12	13.0022	0.01958	0.11748
d13	22.0024	0.01867	0.11202
d14	26.0016	0.01874	0.11244
d15	31.0031	0.01685	0.1011
d23	9.0002	0.02269	0.13614

9. Conclusion

In this work, a step centered on three stages, was presented to calculate dispersions of machining and their influence on the intervals of tolerances. The influence of systematic dispersion accounts for 10% of the total discrepancies under the conditions normal and between 25% and 35%, if the parameters of cut or the cutting tool are badly selected.

The relative value of 10% of the tolerance is very important especially in work in series; because the wear of the tool influences the dimensions of adjustment. An error about the micron influences the overall costs of the end product and risk to guarantee the competitiveness of the product on the market.

Two models of compensation the error in the tool machine numerical control were developed, the first models it is the calculation of systematic dispersion according to the machined length; for the second it is the calculation of the tolerances of manufacture (total dispersions) according to the machined length. These models allowed us optimize the manufacturing dimensions , that is to say by integration in the command balls or in the machining programming

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