JJMIE

Volume 4, Number 5, November 2010
ISSN 1995-6665
Pages 653 - 664

Jordan Journal of Mechanical and Industrial Engineering

Design Optimization of Complex Structures Using Metamodels

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Abstract

Current engineering analyses rely on running expensive and complex computer codes. Statistical techniques are widely used in engineering design to construct approximate models of these costly analysis codes. These models referred as metamodels, are then used in place of the actual analysis codes to reduce the computational burden of engineering analyses. The intent of this study is to provide a comprehensive discussion of the fundamental issues that arise in design optimization using metamodels, highlighting concepts, methods, techniques, as well as practical implications. The paper addresses the selection of design of experiments, metamodel selection, sensitivity analysis and optimization.

Keywords: Metamodel, Experimental design, Central Composite Design, Optimization.

1. Introduction

Traditional engineering design optimization which is the process of identifying the right combination of product parameters is often done manually, time consuming and involves a step by step approach. Approximation methods are widely used to reduce the computational burden of engineering analyses. The use of long running computer simulations in design leads to a fundamental problem when trying to compare and contrast various competing options. It is also not possible to analyze all of the combinations of variables that one would wish. This problem is particularly acute when using optimization schemes. Metamodels, also referred as surrogate models, are a cheaper alternative to costly analysis tools and can significantly reduce the computational time involved. Modern optimization techniques like Genetic Algorithms (GA) have been found to be very robust and general for solving engineering design problems. Evolutionary algorithms such as GA have been used with metamodels (surrogate models) to reduce the cost of exact function evaluations. In this paper, a methodology of developing metamodel and applying it to the optimization problem is explained. As a case study, the roof slab of a Prototype Fast Breeder Reactor was taken and design optimization was carried out. In this approach, experimental design, metamodels, evolutionary algorithm, and finite element analysis tool are brought together to provide an integrated optimization system.

Metamodeling involves (a) choosing an experimental design for generating data, (b) choosing a model to represent the data, and (c) fitting the model to the observed data. There are several options for each of these steps, which will be discussed below. Forrester et. al [1] discussed the recent advances in surrogate based design for global optimization. Simpson et.al [2] has done a survey on the application of metamodels on design. The paper also gives the following recommendations: (i) If many factors(more than 50) must be modeled in a deterministic application, neural networks may be the best choice (ii) If the underlying function to be modeled is deterministic and highly nonlinear in a moderate number of factors (less than 50, say), then kriging may be the best choice despite the added complexity, (ii) In deterministic applications with a few fairly well behaved factors, another option for exploration is using the standard Response surface methodology approach. In Simpson, et al. [3], kriging methods are compared against polynomial regression models for the multidisciplinary design optimization of an aero spike nozzle. Alam et al [4] investigated the effects of experimental design on the development of artificial neural networks as simulation metamodels. This paper shows that a modified-Latin Hypercube design, supplemented by domain knowledge, could be an effective and robust method for the development of neural network simulation metamodels. Queipo et.al. [5] discussed the fundamental issues that arise in the SBAO of computationally

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expensive models such as those found in aerospace systems. The paper mainly focused on the design of experiments based on Latin Hypercube Sampling (LHS) & Orthogonal Arrays (OA) and Surrogate modeling techniques based on polynomial regression model, kriging and radial basis function. Ruichen et.al [6] compares four popular metamodeling techniques— Polynomial Regression, Multivariate Adaptive Regression Splines, Radial Basis Functions, and Kriging— based on multiple performance criteria using fourteen test problems representing different classes of problems. Giunta, et al. [7] also compare kriging models and polynomial regression models for two 5 and 10 variable test problems. In Varadarajan, et al. [8], Artificial Neural Network (ANN) methods are compared with polynomial regression models for the engine design problem in modeling the nonlinear thermodynamic behavior. In Yang, et al., (9), four approximation methods— enhanced Multivariate Adaptive Regression Splines (MARS), Stepwise Regression, ANN, and the Moving Least Square— are compared for the construction of safety related functions in automotive crash analysis, for a relative small sampling size. Similarly many researchers have compared the various experimental designs and/or metamodeling techniques. Only limited researchers are explained about the application of metamodel in the optimization process. This paper explains the methodology of performing experimental design, creating metamodel and applying it to the optimization.

1.1. Design of Experiments Techniques

Design of Experiments includes the design of all information-gathering exercises where variation is present, usually under the full control of the experimenter. Often the experimenter is interested in the effect of some process or intervention on some objects. Design of experiments is a discipline that has very broad application. In the following part, we will introduce the most frequently used DOE techniques.

1.1.1. Full-factorial Design

A full-factorial design is one in which all combinations of all factors at all levels are evaluated. It is an old engineering practice to systematically evaluate a grid of points, requiring $n_1*n_2*n_3*...n_i$ (*i* is the number of factors, *n* is the number of levels for factor *i)* design point evaluations. This practice provides extensive information for accurate estimation of factor and interaction effects. However, it is often deemed cost-prohibitive due to the number of analyses required.

1.1.2. Orthogonal Arrays

The use of orthogonal arrays can avoid a costly full-factorial experiment in which all combinations of all factors at different levels are studied. A fractional factorial experiment is a certain fractional subset (1/2, 1/4, 1/8, etc.) of the full factorial set of experiments, carefully selected to maintain orthogonality (independence) among the various factors and certain interactions. While the use of orthogonal arrays for fractional factorial design suffers from reduced resolution in the analysis of results (i.e., factor effects are aliased with interaction effects as more factors are added to a given array), the significant reduction in the required number of experiments can often justify this loss in resolution as long as some of the interaction effects are assumed negligible. In fractional factorial designs, the number of columns in the design matrix is less than the number necessary to represent every factor and all interactions of those factors. Instead, columns are "shared" by these quantities, an occurrence known as confounding. Confounding results in the dilemma of not being able to realize which quantity in a given column produced the effect on the outputs attributed to that column. In such a case, the designer must make an assumption as to which quantities can be considered insignificant (typically the highest-order interactions) so that a single contributing quantity can be identified.

1.1.3. Latin Hypercube Design

Another class of experimental design which efficiently samples large design spaces is Latin Hypercube sampling. With this technique, the design space for each factor is uniformly divided (the same number of divisions (n*)* for all factors). These levels are then randomly combined to specify n points defining the design matrix (each level of a factor is studied only once). An advantage of using Latin Hypercubes over Orthogonal Arrays is that more points and more combinations can be studied for each factor. The Latin Hypercube technique allows the designer total freedom in selecting the number of designs to run (as long as it is greater than the number factors). While, the configurations are more restrictive using the Orthogonal Arrays. A drawback to the Latin Hypercubes is that, in general, they are not reproducible since they are generated with random combinations. In addition, as the number of points decreases, the chances of missing some regions of the design space increases.

1.1.4. Central Composite Design

Central Composite Design (CCD) is a statistically based technique in which a 2-level full-factorial experiment is augmented with a center point and two additional points for each factor (star points). Thus, five levels are defined for each factor, and to study n factors using Central Composite Design requires $2n + 2n + 1$ design point evaluations. The corner points are for the assessment of linear and 2-way interaction terms. Center points are used to detect curvature and sometime replicated in experimental DOE to estimate pure error. Star points are for the assessment of quadratic terms. Although Central Composite Design requires a significant number of design point evaluations, it is a popular technique for compiling data for Response Surface Modeling due to the expanse of design space covered, and higher order information obtained.

1.1.5. Box-Behnken Design

Box and Behnken developed a family of efficient three-level designs for fitting second-order response surfaces. It exists only for 3-7 factors. Number of runs is very close to CCD for the same number of factors. The Box-Behnken design doesn't have any corners and it is suitable for the situation when corners are not feasible (physical designs).

1.2. Approximating Methods

Approximation concepts were introduced in structural design optimization in the late 1970s to do the following:

- Reduce the number of independent design variables through design variable linking and reduced basis vectors concepts.
- Perform constraint deletion through truncation and regionalization schemes.
- Reduce the number of computer intensive, detailed analyses (or simulation code evaluations) through the use of mathematical approximations of the design optimization objective and constraint functions.

These approximations models can be used to reduce simulation codes or analyses that are computation intensive. They can also help to eliminate the computational noise for simulation codes in the case the outputs rapidly oscillate with gradual changes in the values of input parameters. Computational noise has a strong adverse effect on optimization by creating numerous local optima. Approximation models (Response Surface Models in particular) naturally smooth out the response functions, and, in many cases, help to converge to a global optimum faster. The usage of approximation is not restricted to optimization. It also provides an efficient means of post-optimization or sensitivity analysis. Their value is very high for computationally expensive engineering methods, such as Monte Carlo Simulation, Reliability-Based Optimization, or Probabilistic Design Optimization.

1.2.1. Response Surface Method

Response surface method is a collection of statistical and mathematical techniques useful for developing, improving, and optimizing processes. In some systems based on the underlying engineering, chemical, or physical principles, the nature of the relationship between y and x's might be know exactly. Then a model of the form $y=g(x_1,x_2,\ldots,x_k)+e$ can be written. This type of relationship is often called a mechanistic model. However, the more common situation would be that the underlying mechanism is not fully understood, and the experimenter must approximate the unknown function g with an appropriate empirical $y = f(x_1, x_2, \ldots, x_3) + e$. Usually the function f is a first-order or second-order polynomial. This empirical model is called a response surface model. The model then can be used in optimization studies with a very small computational expense, since evaluation only involves calculating the value of a polynomial for a given set of design variables. Accuracy of the model is highly dependent on the amount of information collected for its construction (number of exact analyses), shape of the exact response function being approximated (like the order of polynomial), and volume of the design space in which the model is constructed (the range covered by the RSM). In a sufficiently small volume of the design space, any smooth function can be approximated by a quadratic polynomial with good accuracy. For highly non-linear functions, polynomials of 3^{rd} or 4^{th} order can be used. If the model is used outside of the design space where it was constructed, its accuracy is impaired, and refining of the model is required. The response surface model relies on the fact that the set of designs on which it is based is well chosen. Randomly chosen designs may cause an inaccurate surface to be constructed or even prevent the ability to construct a surface at all. Because simulations are often time-consuming or the experiments are expensive, the overall efficiency of the design process relies heavily on the appropriate selection of a design set on which to base the approximations. CCD design, Box-Behnken design and D-optimal design are the widely used DOE methods to generate the design set for constructing a response surface model.

1.2.2. Kriging Meta Models

Kriging (named after the South-African mining engineer Krige) is an interpolation method that predicts unknown values. More precisely, a Kriging prediction is a weighted linear combination of all output values already observed. These weights depend on the distances between the new and the observed inputs. The closer the inputs, the bigger the weights are. Kriging models are extremely flexible due to the wide range of correlation functions which can be chosen for building the approximation model. Furthermore, depending on the choice of the correlation function, the model either can provide an exact interpolation of the data, or an inexact interpolation. The most popular DOE for Kriging is Latin Hypercube Design (LHS). LHS offers flexible design sizes n (number of scenarios simulated) for any value of k (number of simulation inputs). Geometrically, many classic designs consist of corners of k-dimensional cubes, so these designs imply simulation of extreme scenarios. LHS, however, has better space filling properties.

1.2.3. Neural Networks

Artificial Neural Networks (ANN) has been studied for many years in the hope of mimicking the human brain's ability to solve problems that are ambiguous and require a large amount of processing. Human brains accomplish this data processing by utilizing massive parallelism, with millions of neurons working together to solve complicated problems. Similarly, ANN models consists of many computational elements, called "neurons" to correspond to their biological counter-parts, operating in parallel and connected by links with variable weights. These weights are adapted during the training process, most commonly through the back-propagation algorithm, by presenting the neural network with examples of input-output pairs exhibiting the relationship the network is attempting to learn. The most common applications of ANN involve approximation and classification. Approximation models attempt to estimate input-output transformation functions, while classification involves using the known inputs to determine class membership. There is no much literature about the optimal experimental design for neural networks or even verification of the effectiveness of the traditional regression model based optimal design methods on the neural net.

2. Methodology

During the optimization process, the model of the component to be optimized will be called for analysis several times, each time with different geometric parameters. So the model has to be in parametric form, which enables it to change the parameter whenever required. So a parametric model of the component has to be modeled using CAD tool which is compatible with the analysis (CAE) tool. Sensitivity analysis of the component was performed to find the effect of the objective function and the state variables (stress/deformation) on the variation of geometric parameters. The parameters which influence more on the state variables are alone considered for the optimization study. In order to reduce the computation cost and to have a better sampling search in the design space, design of experiments was performed using Central Composite Design (CCD). For the sampling points, the computer experiment was conducted using ANSYS package and the results are fed to Minitab software to create the metamodel. This metamodel was used in Genetic Algorithm (GA) coding for optimization.

3. CASE STUDY

The foremost step in the metamodel based optimization is the development of metamodel. Development of metamodel requires lot of experiments to be carried out to the train the model. Experiments may not be feasible in case of complex problems like our case study and in such situations, simulation will be useful. This method of using computer simulation for developing metamodel is termed as design of computer experiments and is explained in detail in the following chapters.

3.1. Parametric Modeling and finite element analysis

As explained earlier, metamodel development requires lot of simulations, for which parametric model of the structure being optimized is required. The structure considered for the metamodel based optimization is a roof slab of a nuclear reactor. The roof slab acts as a support for various components of the reactor and is shown in Figure 1. The main objective of the optimization is to minimize the total weight of the roof slab. As the model will be explored during analysis for various combinations of parameters, a parametric model of the roof slab was developed. The variables taken for parametric modeling are various plate thicknesses and height of the roof slab.

The parametric model was created using the finite element software ANSYS. The necessary loading conditions (weight of various components on the roof slab) and boundary conditions are applied on the structure and a methodology of analyzing the structure for static loading condition was established.

3.2. Sensitivity analysis

The next step in metamodel based optimization is to predict the decision variables for the roof slab through an investigation of the sensitivity of the objective function on small increments of these variables. The design variables considered for the sensitivity analysis are Height (H1), Top and Bottom plate thickness (T_1), Inner shell thickness (T_3), Outer shell thickness (T_4), Stiffener thickness (T_5) and Intermediate Heat Exchanger (IHX), Primary Sodium Pump (PSP) shell thickness (R_1). Sensitivity analysis is carried out using ANSYS sweep optimization module and the analysis reveals that deformation is sensitive to the variations in the parameters H_1, and T_1, stress is sensitive to the variations in the parameters T_1, T_3, T_4, T_5 and R_1, and cost of the roof slab is sensitive to the variations in the parameters T_1 and T_4. So each parameter is contributing to in different aspects and hence all the parameters are taken as design variables for the optimization process. Figure 2 to 5 shows the sensitivity of the objective function (cost) and the state variables (stress and deformation) to the variation of the design variables.

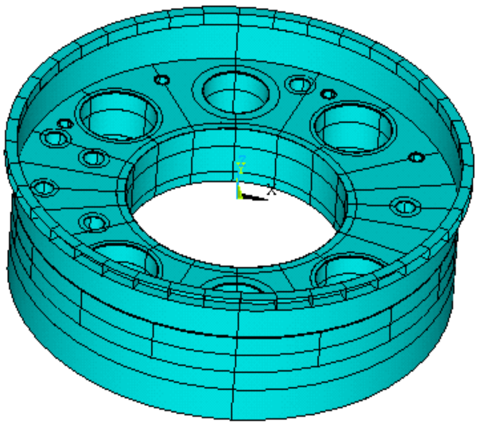

Figure 1. Parametric model of the roof slab

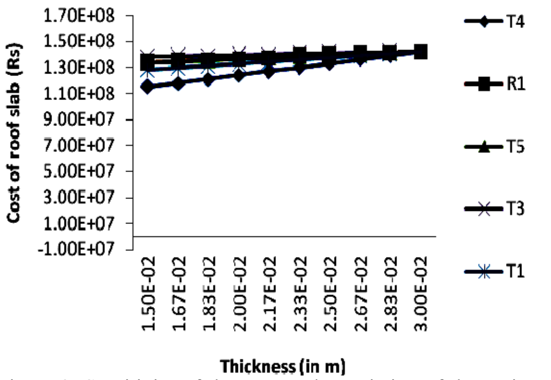

Figure 2. Sensitivity of the cost to the variation of the various thicknesses

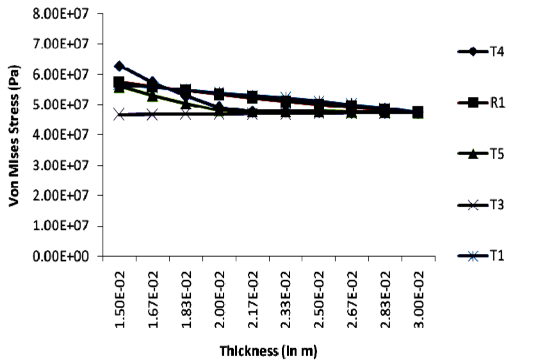

Figure 3. Sensitivity of the maximum stress developed in the roof slab to the variation of the various thicknesses

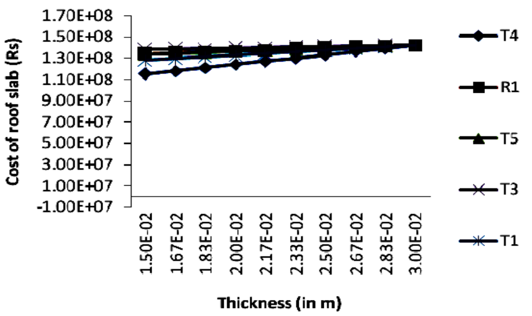

Figure 4. Sensitivity of the maximum deformation on the roof slab to the variation of the various thicknesses

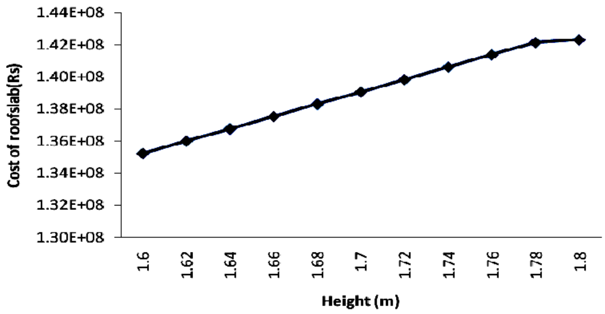

Figure 5. Sensitivity of the cost of the roof slab to the variation of the roof slab height

3.3. Experimental Design

An important issue to metamodeling is to achieve good accuracy of metamodels with a reasonable number of sample points. Experimental design is the sampling plan in design space. The type of experimental design adopted in this work was Central Composite Design (CCD), since many researchers have used this technique for the design of computer experiments [10, 11, 12]. Minitab software has been used to perform the experimental design. The factor H_1 has four levels and factors T_1, T_3, T_4, T_5 and R_1 have two levels each as given in Table 1. Table 2 shows the sample design points based on CCD.

Table 1. Various parameters considered for the optimization of roof slab

Factors	Levels			
	1	2	3	4
H_1(m)	1.8	1.6	-	-
T_1(m)	1.50	2.00	2.25	3.00
T_3(m)	1.50	2.00	2.25	3.00
T_4(m)	1.50	2.00	2.25	3.00
T_5(m)	1.50	2.00	2.25	3.00
R_1(m)	1.50	2.00	2.25	3.00

Table 2. Experimental design sample points based on CCD

H1 m	**T1** x 10^{-2} m	**T3** x 10^{-2} m	**T4** x 10^{-2} m	**T5** x 10^{-2} m	**R1** x 10^{-2} m
1.7	2.25	2.25	2.25	2.25	2.25
1.41	2.25	2.25	2.25	2.25	2.25
1.7	2.25	2.25	2.25	2.25	2.25
1.98	2.25	2.25	2.25	2.25	2.25
1.7	0.13	2.25	2.25	2.25	2.25
1.7	2.25	2.25	2.25	2.25	2.25
1.7	4.37	2.25	2.25	2.25	2.25
1.7	2.25	1.29	2.25	2.25	2.25
1.7	2.25	2.25	2.25	2.25	2.25
1.7	2.25	4.37	2.25	2.25	2.25
1.7	2.25	2.25	0.129	2.25	2.25
1.7	2.25	2.25	2.25	2.25	2.25
1.7	2.25	2.25	4.37	2.25	2.25
1.7	2.25	2.25	2.25	1.29	2.25
1.7	2.25	2.25	2.25	2.25	2.25
1.7	2.25	2.25	2.25	4.37	2.25
1.7	2.25	2.25	2.25	2.25	0.129
1.7	2.25	2.25	2.25	2.25	2.25
1.7	2.25	2.25	2.25	2.25	4.37
1.6	1.50	1.50	1.50	1.50	1.50
1.8	1.50	1.50	1.50	1.50	1.50
1.6	3.00	1.50	1.50	1.50	1.50
1.8	3.00	1.50	1.50	1.50	1.50
1.6	1.50	3.00	1.50	1.50	1.50
1.7	2.25	2.25	2.25	2.25	2.25
1.8	1.50	3.00	1.50	1.50	1.50
1.6	3.00	3.00	1.50	1.50	1.50
1.8	3.00	3.00	1.50	1.50	1.50
1.6	1.50	1.50	3.00	1.50	1.50
1.8	1.50	1.50	3.00	1.50	1.50
1.6	3.00	1.50	3.00	1.50	1.50
1.7	2.25	2.25	2.25	2.25	2.25
1.8	3.00	1.50	3.00	1.50	1.50
1.6	1.50	3.00	3.00	1.50	1.50
1.8	1.50	3.00	3.00	1.50	1.50
1.6	3.00	3.00	3.00	1.50	1.50
1.8	3.00	3.00	3.00	1.50	1.50
1.6	1.50	1.50	1.50	3.00	1.50
1.8	1.50	1.50	1.50	3.00	1.50
1.7	2.25	2.25	2.25	2.25	2.25
1.6	3.00	1.50	1.50	3.00	1.50
1.8	3.00	1.50	1.50	3.00	1.50
1.6	1.50	3.00	1.50	3.00	1.50
1.8	1.50	3.00	1.50	3.00	1.50
1.6	3.00	3.00	1.50	3.00	1.50
1.8	3.00	3.00	1.50	3.00	1.50
1.6	1.50	1.50	3.00	3.00	1.50

1.8	1.50	1.50	3.00	3.00	1.50
1.6	3.00	1.50	3.00	3.00	1.50
1.7	2.25	2.25	2.25	2.25	2.25
1.8	3.00	1.50	3.00	3.00	1.50
1.6	1.50	3.00	3.00	3.00	1.50
1.8	1.50	3.00	3.00	3.00	1.50
1.6	3.00	3.00	3.00	3.00	1.50
1.8	3.00	3.00	3.00	3.00	1.50
1.6	1.50	1.50	1.50	1.50	3.00
1.8	1.50	1.50	1.50	1.50	3.00
1.6	3.00	1.50	1.50	1.50	3.00
1.7	2.25	2.25	2.25	2.25	2.25
1.8	3.00	1.50	1.50	1.50	3.00
1.6	1.50	3.00	1.50	1.50	3.00
1.8	1.50	3.00	1.50	1.50	3.00
1.6	3.00	3.00	1.50	1.50	3.00
1.8	3.00	3.00	1.50	1.50	3.00
1.6	1.50	1.50	3.00	1.50	3.00
1.8	1.50	1.50	3.00	1.50	3.00
1.6	3.00	1.50	3.00	1.50	3.00
1.8	3.00	1.50	3.00	1.50	3.00
1.6	1.50	3.00	3.00	1.50	3.00
1.7	2.25	2.25	2.25	2.25	2.25
1.8	1.50	3.00	3.00	1.50	3.00
1.6	3.00	3.00	3.00	1.50	3.00
1.8	3.00	3.00	3.00	1.50	3.00
1.6	1.50	1.50	1.50	3.00	3.00
1.8	1.50	1.50	1.50	3.00	3.00
1.6	3.00	1.50	1.50	3.00	3.00
1.8	3.00	1.50	1.50	3.00	3.00
1.7	2.25	2.25	2.25	2.25	2.25
1.6	1.50	3.00	1.50	3.00	3.00
1.8	1.50	3.00	1.50	3.00	3.00
1.6	3.00	3.00	1.50	3.00	3.00
1.8	3.00	3.00	1.50	3.00	3.00
1.6	1.50	1.50	3.00	3.00	3.00
1.7	2.25	2.25	2.25	2.25	2.25
1.8	1.50	1.50	3.00	3.00	3.00
1.6	3.00	1.50	3.00	3.00	3.00
1.8	3.00	1.50	3.00	3.00	3.00
1.6	1.50	3.00	3.00	3.00	3.00
1.8	1.50	3.00	3.00	3.00	3.00
1.6	3.00	3.00	3.00	3.00	3.00

3.4. Metamodeling

Metamodeling, often referred as Response Surface Methodology (RSM), involves (a) choosing an experimental design for generating data, (b) choosing a model to represent the data, and (c) fitting the model to the observed data. Detailed description of the RSM is given in Simpson et. al. [2]. Based on the experimental design, the computer experiments were conducted for the various combinations of factors at different levels using the CCD experimental design. The metamodeling technique used in this study is polynomial regression and has been applied by a number of researchers [2, 9, 10, 11, 13] in designing complex engineering systems. The most widely used response surface approximating functions are low-order polynomials. For significant curvature, a second order polynomial which includes all two-factor interactions can be used. A second order polynomial model can be expressed as:

$$\hat{y} = \beta_0+\beta_1 x_1+\beta_2 x_2+\ldots +\beta_k x_k+\beta_{12} x_1 x_2+ \ldots +\beta_{k-1,k} x_{k-1} x_k+ \beta_{11} x_1^2 + \beta_{22} x_2^2 + \ldots + \beta_{kk} x_k^2 \quad (1)$$

The parameters of the polynomial in Equations (1) are usually determined by least squares regression analysis by fitting the response surface approximations to existing data. For the roof slab optimization problem, three metamodels are created to approximate the cost of roof slab, stress developed and deflection using CCD computer experimentation. In order to validate the metamodel some random experiments ware conducted and compared with the finite element simulation of the actual model. The regression coefficients for the three metamodel developed was given in Table 3. Table 4 shows the fitness of the metamodels. Validated regression models of the three responses generated are shown below.Table 3. Regression coefficients for the metamodels.

Table 3. Regression coefficients for the metamodel

Regression coefficients	COST	DMAX	SMAX
β_0	-6.66E+07	3.26E-02	6.97E+08
β_1	1.03E+08	-1.71E-02	-2.77E+08
β_2	1.22E+09	-5.67E-01	-1.12E+10
β_3	4.04E+07	-1.92E-02	-7.01E+06
β_4	6.40E+08	-1.36E-01	-3.80E+09
β_5	-1.95E+08	-8.67E-02	-6.76E+09
β_6	-4.60E+07	-1.12E-01	-8.42E+08
β_7	6.19E+07	1.47E-01	-4.05E+08
β_8	1.99E+08	-5.09E-02	-5.10E+07
β_9	2.16E+08	8.08E-02	1.08E+09
β_{10}	3.83E+08	6.83E-02	2.32E+09
β_{11}	3.08E+08	8.16E-02	5.57E+08
β_{12}	-2.34E+09	1.74E+00	7.65E+09
β_{13}	1.03E+08	-2.62E-01	-8.73E+10
β_{14}	-8.97E+08	-7.51E-01	-4.46E+09
β_{15}	6.58E+08	-5.29E-01	-1.01E+09
β_{16}	8.25E+08	9.55E-01	-1.10E+10
β_{17}	-1.62E+09	8.99E-01	-1.06E+10
β_{18}	6.03E+08	9.10E-01	4.04E+09
β_{19}	3.81E+08	-7.57E-01	-6.12E+09
β_{20}	-1.40E+09	-6.79E-01	-6.79E+09
β_{21}	1.38E+09	-8.01E-01	-6.96E+09
β_{22}	-2.98E+07	1.82E-03	4.35E+07
β_{23}	-4.39E+09	5.18E+00	2.64E+11
β_{24}	-1.95E+09	-3.22E-01	3.29E+09
β_{25}	2.26E+10	-2.00E-01	3.65E+10
β_{26}	2.50E+09	-1.11E-01	3.88E+10
β_{27}	2.50E+09	-2.11E-01	3.74E+09

Table 4. Fitness of metamodels

Response parameter	R-Squared value (%)	R-Squared (Adjusted) value (%)
Cost	78.44	77.76
Stress	82.48	74.86
Deformation	82.2	74.45

3.5. Optimization

The objective of this optimization is to minimize the weight of the roof slab. The method of probabilistic search based on evolutionary algorithms was chosen for the present optimization problem. The real-coded genetic algorithm (RCGA) is developed for obtaining the optimal dimensions of the roof slab of PFBR. The code template developed by Deb [14] was used for this purpose. Certain modifications in the algorithm of this program were necessary to apply it for the present study. RCGA is developed for six input variables and two constraints. The RCGA parameters chosen are; crossover probability=0.8, mutation probability=0.2, number of generation=100, and the population size=60. Various thicknesses of the roof slab and the height of the roof slab are considered as the design variables for optimization. The state variables in the optimization are maximum stress and maximum deformation. In this study the maximum stress is the material yield strength and maximum deflection is the permissible axial movement of the control plug. The range of various design variables with respect to the design requirement is:

- H_1 – [1600-1800] mm
- T_1 – [15 - 30] mm
- T_3 – [15 - 30] mm
- T_4 – [15 - 30] mm
- T_5 – [15 - 30] mm
- R_1 – [15 - 30]mm

The limits for the state variables are 128MPa and 4mm for maximum stress and deflection respectively. Optimization of the roof slab was carried out by this approach and the total volume of the roof slab is reduced by 14.6 % and the cost of roof slab is reduced by 41.4%. Table 5 shows the design and state variables after optimization. The optimized roof slab is also checked for its design adequacy under static and dynamic conditions in Finite Element package ANSYS.

Table 5. Results of optimization process

	Optimization Method	H1 (m)	T1 (m)	T3 (m)	T4 (m)	T5 (m)	R1 (m)	COST (in Cores)	Stress (MPa)	Deformation (m)
Optimized roof slab	GA	1.7	0.020	0.020	0.015	0.015	0.015	8.37	92.4	0.0037
Existing roof slab	-	1.8	0.03	0.03	0.03	0.03	0.03	14.3	82.7	0.0024

4. Conclusion

Traditional solution methods for optimizing complex real life engineering problems can be very expensive and often results in sub-optimal solutions. In this paper, an approach to develop metamodel for complex real time problem is presented. As a case study, a roof slab for which design optimization has to be carried out is considered. A metamodel based optimization approach is presented to address expensive computational cost of large FE runs using meta-models. With the proposed strategy of performing computer experiments, creating metamodel and the application of evolutionary algorithms, this optimization methodology can easily be adopted to more complex structural problems.

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