

Reduction of Vibration of Industrial Equipments

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Abstract

Vibration of industrial equipment is the bad factor influencing its production state, working conditions of staff, and job safety. In course of technology development the more and more potent machines are used. It is quite often accompanied by the increase of a vibration level experienced by the equipment is transmitted to the building structures and through the staffs. A model of the production machine has been installed on the vibration dampers. The system of equation has been permitted to evaluate the reduction of the machine vibrations caused by the unbalance movement of its members, thereby, transmitting it onto the floor.

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1. Introduction

The more potent machines become, the more their vibration disturbs people. Vibration is not only harmfully affects an organism, but also hinders fulfillment of working operations, both mentally, and physical. A vibration with frequencies 25-40 and 60-90 Hz is degraded as visual perception. When frequency of vibration is close to the natural frequency of oscillations of the human body, equal about 5 Hz, operating of vibration becomes especially ideal. In different parts of body the natural frequencies make: in pelvic area 4-6 Hz, in abdominal area 4-8 HZ, and the Head 30 Hz. The effect of vibration to a man is shown at fig. 1 [1].

The sources of vibrations of industrial equipment are: impulse applies technological forces, impacts, unbalance of rotated parts, inertia forces of parts with periodic motion. Therefore, the problem of reducing harmful vibrations remains actually permanently; one solution to the problem is the improvement of the kinematics, balancing of the inertia forces, developing of the shock free technological processes. However these actions are not always possible. That is why the other way to reduce the harmful effect of vibration is the vibration insulation of the equipments, installed on building structures.

Vibration insulation is the reduction of the transmission of vibrations which is reached by the installation of pliable members of small stiffness between vibratory units and adjacent structures [2]. As the vibration

absorbers, the springs, rubber and elastic members, pneumatic, hydraulic, combined devices are used.

2. Model of a machine installed on elastic vibration absorbers

The analysis of vibrations of production equipment carried out by many researchers [2, 3, and 4]. Particularly in [5] the outcomes of the analysis of a vibrational system shown at Fig.2 are adduced. This system corresponds to production machine installed on elastic members. It was considered that the machine consists of fixed and moving part. The last ones are the masses m_1, m_2, \dots, m_m , the motion of which is determined by the machine structure which is considered less than the mass M_o of fixed parts. The model is placed in the systems rectangular coordinates. One of them XYZ is connected to mass M_o , origin of the system O is combined with the center of mass G_o . Other ξ_o, η_o, ζ_o , with the origin in the same point O, is independent, fixed in space. The axes of the system XYZ coincide with principal axes of inertia of the mass M_o . In the balanced state both of these systems coincide. The connection of systems at the arbitrary moment is determined, the coordinates ξ_o, η_o, ζ_o , the point G_o and angles φ, ψ, θ , which are selected to be small for small oscillations of the mass M_o . The mass of the i^{th} traveling part is m_i , coordinates of its center of gravity are x_i, y_i, z_i . The sum of parts m , is equal m , its coordinates are X, Y, Z.

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From the expressions for a kinetic energy after transformations we got the left parts of the Lagrange equations of the second kind:

$$\left. \begin{aligned} (M_o + m) \ddot{\xi}_o + m \ddot{X} \\ (M_o + m) \ddot{\eta}_o + m \ddot{Y} \\ (M_o + m) \ddot{\zeta}_o + m \ddot{Z} \\ (A + a) \ddot{\phi} + \sum m_i (\ddot{y}_i x_i - \ddot{x}_i y_i) \\ (B + b) \ddot{\psi} + \sum m_i (\ddot{x}_i z_i - \ddot{z}_i x_i) \\ (C + c) \ddot{\theta} + \sum m_i (\ddot{z}_i y_i - \ddot{y}_i z_i) \end{aligned} \right\} \quad (1)$$

In which:

$m\ddot{X}$, $m\ddot{Y}$ and $m\ddot{Z}$ - Inertia forces of the linearly moving parts,

$$P = \frac{1}{2} \left(\sum_{i=1}^n C_{\xi_i} u_i^2 + \sum_{i=1}^n C_{\eta_i} v_i^2 + \sum_{i=1}^n C_{\zeta_i} w_i^2 + \sum_{i=1}^n K_{\xi_i} \phi^2 + \sum_{i=1}^n K_{\eta_i} \psi^2 + \sum_{i=1}^n K_{\zeta_i} \theta^2 \right) \quad (3)$$

Where:

u, v and w - Deformation of the elastic members,

C - Linear stiffness of elastic members,

K - Torsional stiffness of elastic members.

The deformation of the elastic members can be expressed as follows:

Substitute (4) in (3) we get:

$$P = \frac{1}{2} \left[\sum_{i=1}^n C_{\xi_i} (\xi_o + \eta_i \phi - \zeta_i \psi)^2 + \sum_{i=1}^n C_{\eta_i} (\eta_o + \zeta_i \theta - \xi_i \phi)^2 + \sum_{i=1}^n C_{\zeta_i} (\zeta_o + \xi_i \psi - \eta_i \theta)^2 + \sum_{i=1}^n k_{\xi_i} \theta^2 + \sum_{i=1}^n K_{\eta_i} \psi^2 + \sum_{i=1}^n K_{\zeta_i} \phi^2 \right] \quad (5)$$

Finding of partial derivatives from potential energy on generalized coordinates gives right hand members of the system of differential equations depicting oscillations of

$$\left. \begin{aligned} (M_o + m) \ddot{\xi}_o + (\xi_o C_{\xi} + \phi u_{\eta} - u_{\zeta} \psi) &= -m \ddot{X} \\ (M_o + m) \ddot{\eta}_o + (\eta_o C_{\eta} + \theta v_{\zeta} - v_{\xi} \phi) &= -m \ddot{Y} \\ (M_o + m) \ddot{\zeta}_o + (\zeta_o C_{\zeta} + \psi \omega_{\xi} - \omega_{\eta} \theta) &= -m \ddot{Z} \\ (A + a) \ddot{\phi} + (\phi C_{\zeta\zeta} + \xi_o u_{\eta} - \eta_o v_{\xi} - \psi C_{\eta\zeta} - \theta C_{\zeta\xi}) &= -\sum m_i (\ddot{y}_i x_i - \ddot{x}_i y_i) \\ (B + b) \ddot{\psi} + (\psi C_{\eta\eta} - \xi_o u_{\zeta} + \zeta_o \omega_{\xi} - \theta C_{\xi\eta} - \phi C_{\eta\zeta}) &= -\sum m_i (\ddot{x}_i z_i - \ddot{z}_i x_i) \\ (C + c) \ddot{\theta} + (\theta C_{\xi\xi} + \eta_o v_{\zeta} - \zeta_o \omega_{\eta}) &= -\sum m_i (\ddot{z}_i y_i - \ddot{y}_i z_i) \end{aligned} \right\} \quad (6)$$

$$\sum m_i (\ddot{y}_i x_i - \ddot{x}_i y_i), \quad \sum m_i (\ddot{x}_i z_i - \ddot{z}_i x_i) \text{ and } \sum m_i (\ddot{z}_i y_i - \ddot{y}_i z_i) \text{ - disturbing moments,}$$

A, B, C – principal moments of inertia of the mass M_o concerning fixed axes,

a, b, c – total moments of traveling masses concerning axes XYZ, and they will be expressed as follows:

$$\left. \begin{aligned} a &= I_{1z} + I_{2z} + \dots + I_{iz} \\ b &= I_{1y} + I_{2y} + \dots + I_{iy} \\ c &= I_{1x} + I_{2x} + \dots + I_{ix} \end{aligned} \right\} \quad (2)$$

where I_{ix} , I_{iy} and I_{iz} - moments of inertia of i^{th} mass concerning axes X, Y, Z.

The right hand members of Lagrange differential equations are composed by means of the expressions for the potential energy of the system. The potential energy for n elastic members is given by the following expression:

$$\left. \begin{aligned} u_i &= \xi_o + \eta_i \phi - \zeta_i \psi \\ v_i &= \eta_o + \zeta_i \theta - \xi_i \mu \\ \omega_i &= \zeta_o + \xi_i \psi - \eta_i \theta \end{aligned} \right\} \quad (4)$$

the model of the machine installed on the elastic shock-absorbers [6](Fig.2):

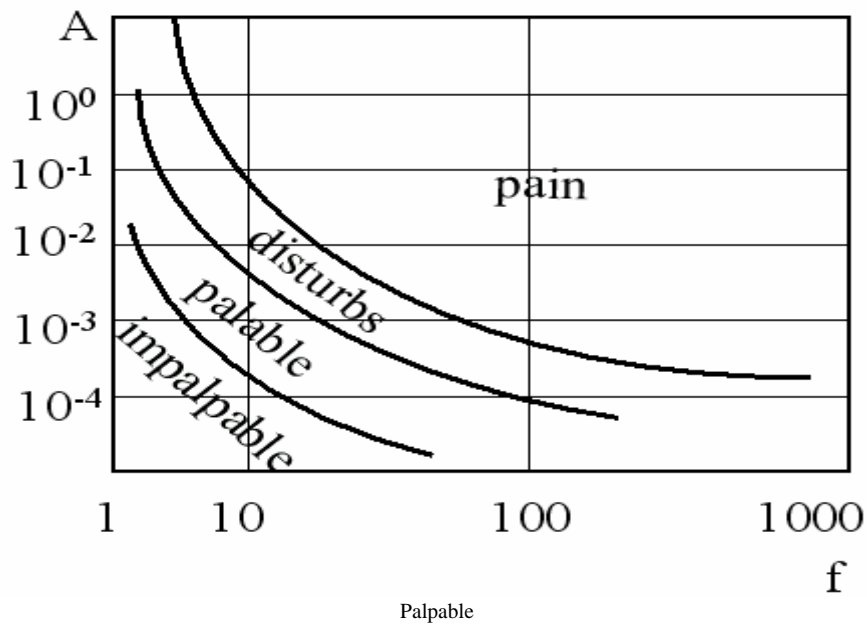


Fig. 1. Effect nature of vibrations on human organism, A-amplitude , mm, f- frequency, Hz

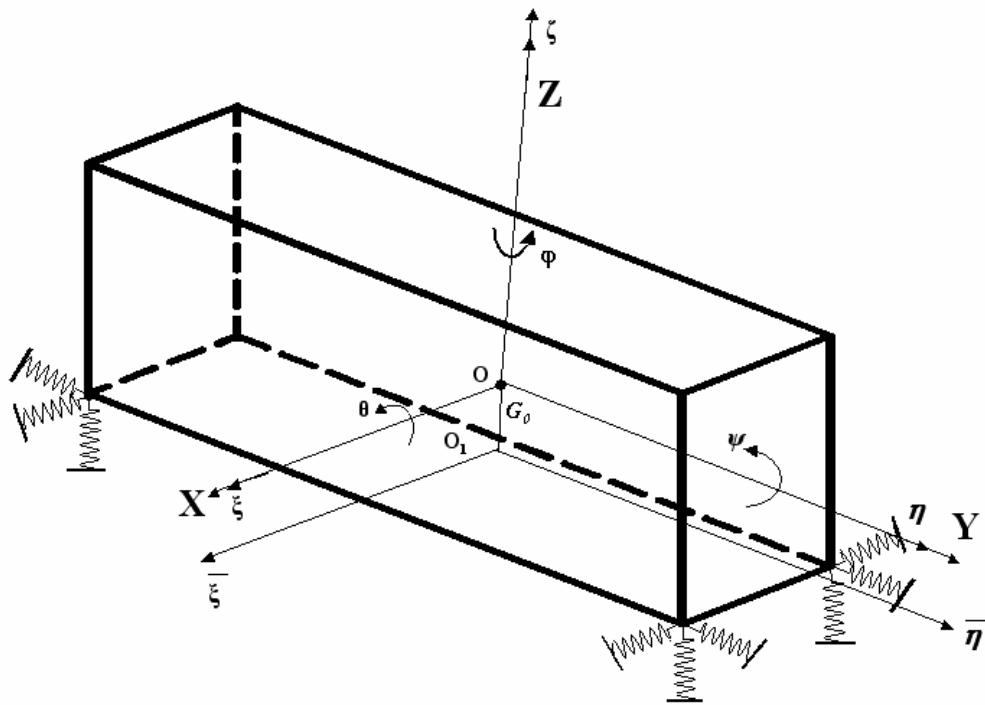


Fig.2. Model of the production machine installed on elastic members

In these equations:

*The linear stiffnesses are:

$$\left. \begin{aligned} C_{\xi} &= \sum_{i=1}^n C_{\xi i} \\ C_{\eta} &= \sum_{i=1}^n C_{\eta i} \\ C_{\zeta} &= \sum_{i=1}^n C_{\zeta i} \end{aligned} \right\} \quad (7)$$

* Linear – rotary stiffnesses are:

$$\left. \begin{aligned} u_{\eta} &= \sum_{i=1}^N C_{\zeta i} \eta_i & u_{\zeta} &= \sum_{i=1}^N C_{\zeta i} \xi_i \\ v_{\xi} &= \sum_{i=1}^n C_{\eta i} \xi_i & v_{\zeta} &= \sum_{i=1}^n C_{\eta i} \zeta_i \\ w_{\xi} &= \sum_{i=1}^n C_{\zeta i} \xi_i & w_{\eta} &= \sum_{i=1}^n C_{\zeta i} \eta_i \end{aligned} \right\} \quad (8)$$

*The torsional stiffnesses are:

$$\left. \begin{aligned} C_{\xi\xi} &= \sum_{n=1}^n (K_{\xi i} + C_{\eta i} \xi_i^2 + C_{\zeta i} \eta_i^2) \\ C_{\eta\eta} &= \sum_{n=1}^n (K_{\eta i} + C_{\zeta i} \zeta_i^2 + C_{\xi i} \xi_i^2) \\ C_{\zeta\zeta} &= \sum_{n=1}^n (K_{\zeta i} + C_{\xi i} \xi_i^2 + C_{\eta i} \eta_i^2) \end{aligned} \right\} \quad (9)$$

* Gyroscopic stiffnesses are:

$$\left. \begin{aligned} C_{\xi\eta} &= C_{\eta\xi} = \sum_{i=1}^n C_{\zeta i} \xi_i \eta_i \\ C_{\eta\zeta} &= C_{\zeta\eta} = \sum_{i=1}^n C_{\xi i} \eta_i \zeta_i \\ C_{\zeta\xi} &= C_{\xi\zeta} = \sum_{i=1}^n C_{\eta i} \zeta_i \xi_i \end{aligned} \right\} \quad (10)$$

The given system includes in its right hand members the disturbing forces caused by the unbalanced moving masses. Equations (6) can be used at the designing of vibration dampers (absorber) in order to avoid loads transmitted by the machine onto the floor.

In many cases not all the motions of the machine are interdependent, and then the system (6) is simplified. If two principal central axes of stiffness are only principal axes of inertia, but not the central ones, the principal central axis of inertia will be the third principal central

axis of stiffness then all gyroscopic (10) and four of six linear – rotary (8) stiffnesses are equal zero. In this case the system (6) is essentially simplified and becomes:

$$\left. \begin{aligned} (M_o + m)\ddot{\xi}_o + \xi_o C_{\xi} &= -m\ddot{X} \\ (M_o + m)\ddot{\eta}_o + \eta_o C_{\eta} &= -m\ddot{Y} \\ (M_o + m)\ddot{\zeta}_o + \zeta_o C_{\zeta} &= -m\ddot{Z} \\ (A + a)\ddot{\varphi} + \varphi C_{\zeta\zeta} &= -\sum m_i (\ddot{y}_i \chi - \ddot{\chi}_i y_i) \\ (B + b)\ddot{\psi} + \psi C_{\eta\eta} &= -\sum m_i (\ddot{\chi}_i z_i - \ddot{z}_i \chi_i) \\ (C + c)\ddot{\theta} + \theta C_{\xi\xi} &= -\sum m_i (\ddot{z}_i y_i - \ddot{y}_i z_i) \end{aligned} \right\} \quad (11)$$

And describe independent oscillations along each axis of coordinates and around it.

3. Developing of the model of the machine installed on the elastic-dissipative vibration dampers

The systems (6) and (11) are not taking into account for the dissipation of the energy of oscillations in the vibration dampers. To take into consideration this dissipation we shall add a damping to the vibration dampers and simplify the scheme a little, having shown only vertical components of each of four hearings [7]. In this case the machine is presented by the model shown at Fig.3.

The origin of the coordinated system is placed in the point of the static equilibrium of the center of masses of the machine, as the axes of coordinates the central axes of inertia of the machine are considered. The model has six degrees of freedom by the way of linear displacement along axes and angular displacements around the last. As a result of an unbalance of mobile masses there is a disturbing force [8, and 9], which can be accepted equal $Q = \sin \omega t$, as well as in the already reviewed model.

The restoring force of each vibration damper is proportional to its deformation. The force of a viscous strength of vibration damper absent in the model at Fig.2 is proportional to the speed of deformation.

The vibrations of the model (Fig, 3) are described by the following equations: (Equ No.12)

$$\left. \begin{aligned} m\ddot{x} &= \sum_{i=1}^4 F_{xi} + Q\sin\omega t \\ m\ddot{y} &= \sum_{i=1}^4 F_{yi} \\ m\ddot{z} &= \sum_{i=1}^4 F_{zi} - P \\ I_x\ddot{\psi} &= -F_{z1}l_{y1} + F_{z2}l_{y2} - F_{z3}l_{y1} + F_{z4}l_{y2} + \sum_{i=1}^4 F_{yi}z_{bi} \\ I_y\ddot{\varphi} &= F_{z1}l_{x2} + F_{z2}l_{x2} - F_{z3}l_{x1} - F_{z4}l_{x1} + \sum_{i=1}^4 F_{xi}z_{bi} - Q\sin\omega t z_o \\ I_z\ddot{\theta} &= F_{x1}l_{y1} - F_{x2}l_{y2} + F_{x3}l_{y1} - F_{x4}l_{y2} + F_{y2}l_{x2} - F_{y3}l_{x1} - F_{y4}l_{x1} \end{aligned} \right\}$$

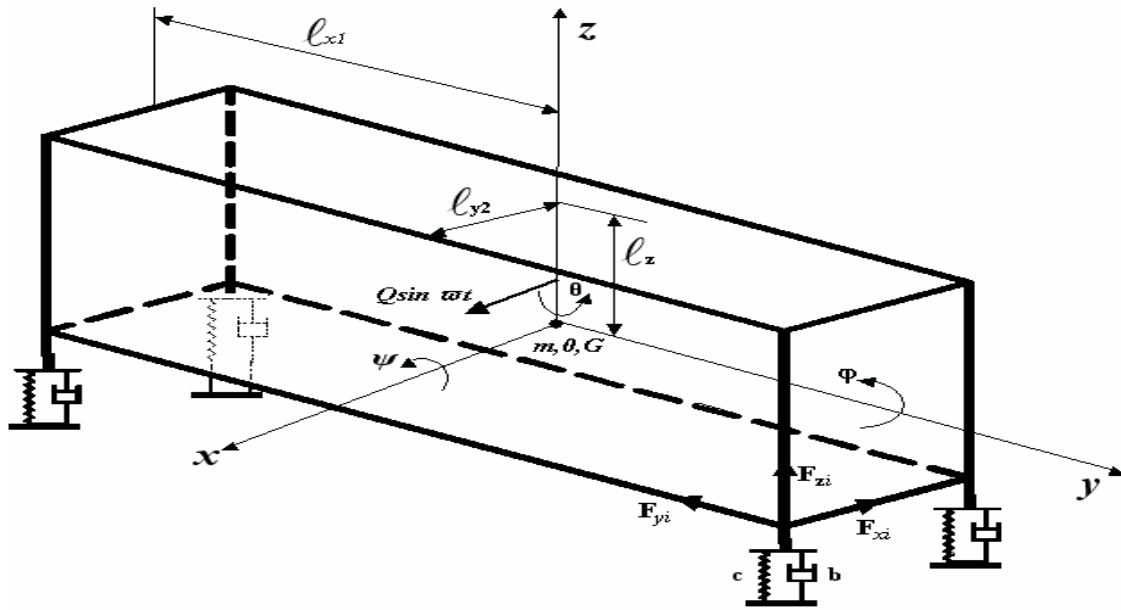


Fig.3. Model of the production machine installed on elastic-damping supports.

Where

$$\left. \begin{aligned}
 F_{x1} = F_{x3} &= -C_x(x + z_b\varphi + l_{y1}\theta) - b_x(\dot{x} + z_b\dot{\varphi} + l_{y1}\dot{\theta}) \\
 F_{x2} = F_{x4} &= -C_x(x + z_b\varphi + l_{y2}\theta) - b_x(\dot{x} + z_b\dot{\varphi} + l_{y2}\dot{\theta}) \\
 F_{y1} = F_{y2} &= -C_y(y + z_b\psi + l_{x2}\theta) - b_y(\dot{y} + z_b\dot{\psi} + l_{x2}\dot{\theta}) \\
 F_{y3} = F_{y4} &= -C_y(y + z_b\psi + l_{x1}\theta) - b_y(\dot{y} + z_b\dot{\psi} + l_{x1}\dot{\theta}) \\
 F_{z1} &= -C_z(z + l_{x2}\varphi + l_{y1}\psi) - b_z(\dot{z} + l_{x2}\dot{\varphi} - l_{y1}\dot{\psi}) \\
 F_{z2} &= -C_z(z + l_{x2}\varphi + l_{y2}\psi) - b_z(\dot{z} + l_{x2}\dot{\varphi} - l_{y2}\dot{\psi}) \\
 F_{z3} &= -C_z(z + l_{x1}\varphi + l_{y1}\psi) - b_z(\dot{z} - l_{x1}\dot{\varphi} - l_{y1}\dot{\psi}) \\
 F_{z4} &= -C_z(z + l_{x1}\varphi + l_{y2}\psi) - b_z(\dot{z} - l_{x1}\dot{\varphi} + l_{y2}\dot{\psi})
 \end{aligned} \right\} \quad (13)$$

The substitution (13) in (12) and the transformations lead to the system of linear differential equations of the second order.

$$\left. \begin{aligned}
 M\ddot{x} + b_{11}\dot{x} + b_{15}\dot{\varphi} + b_{16}\dot{\theta} + c_{11}x + c_{15}\varphi + c_{16}\theta &= Q\sin\omega t \\
 M\ddot{y} + b_{22}\dot{y} + b_{24}\dot{\psi} + b_{26}\dot{\theta} + c_{22}y + c_{24}\psi + c_{26}\theta &= 0 \\
 M\ddot{z} + b_{33}\dot{z} + b_{34}\dot{\psi} + b_{35}\dot{\varphi} + c_{33}z + c_{34}\psi + c_{35}\varphi &= -p \\
 I_x\ddot{\psi} + b_{42}\dot{y} + b_{43}\dot{z} + b_{44}\dot{\psi} + b_{45}\dot{\varphi} + b_{46}\dot{\theta} + c_{42}y + c_{43}z + c_{44}\psi + c_{45}\varphi + c_{46}\theta &= 0 \\
 I_y\ddot{\varphi} + b_{51}\dot{x} + b_{53}\dot{z} + b_{54}\dot{\psi} + b_{55}\dot{\varphi} + b_{56}\dot{\theta} + c_{51}x + c_{55}\varphi + c_{56}\theta &= -Qz_b\sin\omega t \\
 I_z\ddot{\theta} + b_{61}\dot{x} + b_{62}\dot{y} + b_{64}\dot{\psi} + b_{65}\dot{\varphi} + b_{66}\dot{\theta} + c_{61}x + c_{62}y + c_{64}\psi + c_{65}\varphi + c_{66}\theta &= 0
 \end{aligned} \right\} \quad (14)$$

in which stiffnesses C_{ij} are described by the following relations:

$$\left. \begin{aligned}
 c_{11} &= 4c_x; c_{15} = c_{51} = 4c_x z_b; c_{16} = c_{61} = 2c_x (l_{y1} - l_{y2}) \\
 c_{22} &= 4c_y; c_{24} = c_{42} = 4c_y z_b; c_{26} = c_{62} = 2c_y (l_{x2} - l_{x1}) \\
 c_{33} &= 4c_z; c_{34} = c_{43} = 4c_z (l_{y2} - l_{y1}); c_{35} = c_{53} = 2c_z (l_{x2} - l_{x1}) \\
 c_{44} &= 2c_z (l_{y1}^2 + l_{y2}^2) + 4c_y z_b^2; \\
 c_{45} &= c_{54} = c_z (l_{x2} l_{y2} + l_{x1} l_{y1} - l_{x1} l_{y2} - l_{x2} l_{y1}); \\
 c_{46} &= c_{64} = 2c_y z_b (l_{x2} - l_{x1}) \\
 c_{55} &= 2c_z (l_{x1}^2 + l_{x2}^2) + 4c_x z_b^2; c_{56} = c_{65} = 2c_x z_b (l_{y1} - l_{y2}) \\
 c_{66} &= 2c_x (l_{y1}^2 + l_{y2}^2) + 2c_y (l_{x1}^2 + l_{x2}^2) \\
 c_{12} &= c_{13} = c_{14} = 0; c_{23} = c_{25} = 0; c_{36} = 0
 \end{aligned} \right\} \quad (15)$$

The coefficients of strength are described quite similarly.

4. Solution of the composed set of equations

The natural frequencies are found by means of the determinant (16) of the equation system (13) under the condition that for the zero instant at the linear and angular displacements and the speeds at all coordinates are considered equal zero and that the right side members of the equation system (13) also equal zero.

$$\begin{vmatrix}
 c_{11} - M_w^2 & 0 & 0 & 0 & c_{15} & c_{16} \\
 0 & c_{22} - M_w^2 & 0 & c_{24} & 0 & c_{26} \\
 0 & 0 & c_{33} - M_w^2 & c_{34} & c_{35} & 0 \\
 0 & c_{42} & c_{43} & c_{44} - I_x w^2 & c_{45} & 0 \\
 c_{51} & 0 & c_{53} & c_{54} & c_{55} - I_y w^2 & c_{56} \\
 c_{61} & c_{62} & 0 & c_{64} & c_{65} & c_{66} - I_z w^2
 \end{vmatrix} = 0 \quad (16)$$

As an example let us consider the production machine [5] of the mass $m=1500$ kg. Moments of inertia concerning principal axes are $I_x=300$ Nms², $I_z=630$ Nms². Frequency of rotation speed of the main shaft $n=230$ rpm, frequency of the disturbing force $\omega=24$ s⁻¹, its amplitude $Q=5$

$$\begin{aligned}
 c_x &= c_y = 1011 \text{ Nm}^{-1}, \\
 c_z &= 3 \cdot 10^6 \text{ Nm}^{-1}.
 \end{aligned}$$

Damping factors:

$$\begin{aligned}
 b_x &= b_y = 2.5 \cdot 10^6 \text{ Nsm}^{-1}, \\
 b_z &= 4.3 \cdot 10^3 \text{ Nsm}^{-1}.
 \end{aligned}$$

Coordinates of supports:

$$\begin{aligned}
 Z_b &= 0.5 \text{ m}, \\
 \ell_{x1} &= 1.0 \text{ m}, \\
 \ell_{x2} &= 0.54 \text{ m}, \\
 \ell_{y1} &= 1.8 \text{ m}, \\
 \ell_{y2} &= 0.83 \text{ m},
 \end{aligned}$$

The solution of the set of equations for the case of natural vibrations of the machine in a vertical direction found by means of MATLAB, demonstrates, that this natural frequency makes 71 s⁻¹, whereas the disturbing

frequency, as was mentioned, equal 24 s⁻¹. Hence the machine runs in under resonance mode, in which natural frequency is far enough from a resonance.

The calculation, also by means of MATLAB, of the enforced vertical vibrations has shown, that their amplitude $X_0 = 1.2 \cdot 10^{-4}$ m. Then the dimensionless dynamic factor is

$$K_d = cx_o / Q = 0.2.$$

Comparing its value with a unit, we are convinced, that the first one is much less, and we come to the conclusion, that the vibration insulation of the given machine meets the lead requirements [7].

5. Conclusion

1. The model offered herein of the production machine is installed on the vibration dampers and the developed with the account of elastic and dissipative properties of the vibration dampers. The system of equation is permitted to evaluate the reduction of the machine vibrations caused by the unbalance movements of its members and, thereby transmitting it onto the floor.
2. By means of the developed system of equations, it is proven that, taken as an example the production

machine that runs far from a resonance and its vibration dampers effectively meet the requirements of the working environment.

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