

An Automatic Method for Creating the Profile of Supersonic Convergent-Divergent Nozzle

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Abstract:

The supersonic convergent-divergent nozzles have many applications. An advanced design method must be used when the conditions are critical and wave-free stream must be reached. The characteristic method is a general technique in which the design of a nozzle can be achieved. Unfortunately, this technique can be performed only through a lengthy graphical procedure. The aim of this paper is to create an automatic way to perform the nozzle profile using the above method. Although, there are a variety of CFD & FE packages that deal with nozzle design, they all require the profile of the nozzle to be started with. This research develops a new approach that helps the machine designer to create this profile. Through this work, two computational stages have been used and both with the aid of the computer. Firstly, a well-known calculation and graphical packages have been used to create the unit model matrix for each Mach number. Then, in the second stage, programming with visual basic has been used to create the profile of the nozzle automatically by multiplying the unit model matrix by the proper scale factor that is calculated according to the working requirements. User friendly software has been written for this purpose. The functionality of the method is validated by solving problems with known solutions and this approach presented logical results. This work is limited for Mach number up to 2.5 only, constant specific heat and two dimensional flows.

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Keywords: Convergent-divergent supersonic nozzle; Characteristic method; Mach-wave; Prandtl-Meyer angle; maximum expansion angle and algorithm.

Nomenclature

A	Cross sectional area (mm ²)
D	Diameter (mm)
H	Height (mm)
L	Length of the nozzle (mm)
M	Mach number
m	Mass flow rate (kg/s)
N	Speed index = 2θ
P	Pressure (N/m ²)
PDEs	Partial differential equations
R	Gas constant = 8.31434 (kJ/kmol . K)
SCDN	Supersonic convergent divergent nozzle
T	Temperature (°K)
u, V	The Velocity (m/s)
γ	The Ratio of Specific Heats
μ	Mach angle (Degree)
θ	Prandtl-Meyer angle (Degree)
θ _a	Flow inclination(Degree)
ξ	Characteristic Subscripts
i	Inlet
e	Exit
t	Throat

1. Introduction

Supersonic nozzle has many applications in industry. These include, and not limited to, the areas of gas turbines and turbo machinery [1], separation systems [2 and 3], geo-environmental techniques [4], air conditioning [5] and many others. It is used in many ways like: De Laval nozzle in the high-speed steam turbine, in the jet propulsion units, in rocket motors, in the drive of air turbines, in gas turbines, in the thermal ejectors and in the supersonic wind tunnels.

The basic theory (one and two dimensional analysis) of the design of the super sonic convergent divergent nozzle(SCDN) can be found in many references like those at the end of this paper. More advanced methods must be used when the conditions are critical and wave free stream must be reached. The characteristic method is a general technique in which the design of nozzles can be achieved [6]. Unfortunately, this technique can be performed only through a lengthy graphical procedure. Step-by-step graphical procedure must be accurately followed. Although, there is a wide range of CFD and FE packages that deals with nozzle design each of these needs the profile of the nozzle to start with [7,8]. Dimarogonas [7]

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concluded that any attempt to utilize computer methods for machine design or analysis should start with the description of the geometry of the machine or a machine part in a complete and unambiguous manner by a set of computer-stored information that can be further utilized for analysis or design. The aim of this paper is to create an automatic way to perform the profile of the SCDN. To generate such a profile the method of characteristic can be used and the graphical procedure is performed automatically by aid of the computer. Unfortunately, this graphical procedure cannot be programmed directly and a primary stage of data preparing is needed. Microsoft Excel and AutoCAD have been chosen to create the database required. A Visual Basic code has been developed in order to treat with this database and then to determine the required profile.

On other words, the objective of this research is to develop a new method that performs the creation of the profile of the SCDN automatically. A user friendly software package helps the machine designer to get this profile that produces uniform supersonic flow at the end of the nozzle. Input parameters for the software will include for example: air properties, mass flow rate, and the Mach number required at exit. The output of the software will include a plot of the nozzle profile, a table that contains the coordinates of this profile in both x and y directions and a full report in win-word or notepad format. This paper is structured as in the following: Section two covers the theoretical background of the different approaches of modelling. Section three outlines the methodology of the research and the developments of the software. Section four shows the results and discusses them. Section five provides the summary and conclusions of the research.

2. Theoretical Background

The design of the supersonic convergent-divergent nozzle (SCDN) has different approaches with different levels of simplifications. The following sections will cover these levels.

2.1 One-Dimensional Flow :

The concept of 'one-dimensional' flow in any form of conduit is that all relevant quantities (velocity, pressure, density and so on) are constant over any cross-section of that conduit. Thus the flow can be described in terms of only one coordinate and time. The flow of a real fluid is never strictly one-dimensional because of the presence of boundary layers, but the assumption provides satisfactory solutions of many problems in which the boundary layer is not very thick and there are no abrupt changes of cross-section. Many references in the literature [e.g. 9] use the one-dimensional idealization to examine instances of flow in which the effects of compressibility are of particular importance. The whole analysis will not be repeated here and only some important points will be drawn. The main relation between the area and velocity is

$$dA/A = du/u (M^2-1) \quad (1)$$

Several important conclusions may be drawn from equation (1). For subsonic velocities ($M < 1$), dA and du must be opposite in sign. That is, an increase of cross-sectional area causes a reduction of velocity and vice

versa. For supersonic velocities, however, M^2-1 is positive and so dA and du are of the same sign. When $M=1$ or $du=0$, dA must be zero and A must be a minimum.

The relation between throat area, and choked mass flow rate for air, with constant specific heat is

$$m = \rho AV = 0.686 A_t P_i / (R T_i)^{1/2} \quad (2)$$

This equation shows that no matter how much the exit pressure is reduced, or how the shape of the duct may change upstream or downstream of the minimum cross-section, this maximum flow rate cannot be exceeded. The nozzle will be either subsonic throughout, over-expanding or under-expanding depending on the exit pressure. (For more details see [10 and 11].

2.2 Multidimensional Flow:

The computation of exact or nearly exact flows of a frictionless incompressible fluid around general two-dimensional boundaries offers problems of some mathematical difficulty [12, 13, 14 and 6]. The additional condition that the density of the fluid may vary will complicate the problem under certain conditions; however, in those cases where the velocities in the flow field are everywhere greater than the local speed of sound, a fundamental simplification occurs. James [15], presented the nonlinear partial differential equations (PDEs) of motions (continuity and momentum) for multidimensional flow. He showed that direct integration of these equations over the entire flow field is difficult unless further assumptions are made about the nature of the flow. The mathematics associated with a system of nonlinear PDEs is extremely complex; in many cases, solutions are not possible. To reduce the equations to more workable form, one shall place certain restrictions on the flow and thereby linearize the PDEs. In certain cases, as the case of the SCDN, however, approximate solutions obtained by linearization are not adequate, but greater accuracy is necessary. Instead of dropping terms, it is often desirable to consider the complete nonlinear equation. Fortunately, as James maintained, the very nature of supersonic flow allows a numerical method of solution of the complete differential equation for problems in two-dimensional flow, axisymmetric flow, and one dimensional unsteady flow. This process is known to mathematicians as the "method of characteristics".

2.3 Method of Characteristics:

The method of characteristic is a general technique in which partial differential equations of pressure transients are converted into particular simultaneous total differential equations which, after being expressed in finite-difference form, can be solved by computer. The fundamental idea of the method of characteristics is to replace the infinite number of waves by a finite number which can then be treated separately. In the areas between waves, the velocity magnitude and direction are considered constant. Therefore it is necessary to replace the actual boundaries of the flow by approximate boundaries with a finite number of disturbances. In other words, a curved boundary is replaced by a number of straight-line segments, with a definite angle between them [6]. The process of computing the flow change from region to another is thus reduced to the purely mechanical process. This approach is explained

in details in the work of [3 and 6] and will not be repeated here and only some important relations will be given.

A characteristic is a line that exists only in supersonic flows. Characteristics should not be confused with finite strength waves, such as shock waves [13]. The ξ characteristic is inclined to the local streamline by the angle μ , which is the Mach angle,

$$\mu = \sin^{-1}(1/M) \quad (3)$$

The difference between a shock wave and a Mach wave should be kept in mind. A Mach wave represents a surface across which some derivative of the flow variables (such as the thermodynamic properties of the fluid and the flow velocity) may be discontinuous while the variables themselves are continuous. A shock wave represents a surface across which the thermodynamic properties and flow velocity are essentially discontinuous. Thus, the characteristic curves, or Mach lines, are patching lines for continuous flows, whereas shock waves are patching lines for discontinuous flows. The use of the isentropic relationships then permits an evaluation of the change of pressure and temperature taking place through the expansion. At a concave corner in supersonic flow, an isentropic expansion takes place [15]. This flow consists of a large number of expansion Mach waves; across each wave the changes in flow properties are infinitesimally small. The resultant flow is analyzed by using the equations of continuity, momentum, and energy for a perfect gas with constant specific heats. The sum of the angles of the concave wall ($\Sigma\theta_a$) that called the Prandtl-Meyer angle expansion (θ) is a function of the Mach number as given by

$$\theta = \sqrt{\frac{y+1}{y-1}} \tan^{-1} \sqrt{\frac{y-1}{y+1} (m^2 - 1)} - \tan^{-1} \sqrt{m^2 - 1} \quad (4)$$

Equation (4) above has been derived in details by James [15]. The result of this analysis has been presented in tabular form, showing the change in Mach number occurring for a given flow turning angle. The use of the isentropic relationships then permits an evaluation of the change of pressure, temperature, and other thermodynamics taking place through the expansion. The maximum permissible expansion angle (θ_{max}) for any nozzle design, as Puckett [6] maintained, is exactly one half of the Prandtl-Meyer angle expansion (θ).

When a Prandtl-Meyer expansion flow impinges on a plane wall, sufficient waves must be generated to maintain the wall boundary condition; that is, at the wall surface, the flow must be parallel to the wall. To cancel the incident wave, the wall must turn through an angle of θ_a at the point of impingement of the incident wave. The resultant wave interactions present complexities that render an exact analysis of the flow which is extremely difficult; however, the general nature of the flow can be recognized. There is no reflected wave, since the boundary condition at the wall is satisfied without it; the incident wave is effectively canceled. One more use of this idea is the nature of effect of the centerline. Since, from symmetry there can be no flow across the center streamline; this streamline can be replaced by a plane wall. This fact will allow the designer

to reduce the calculation as only one side of the nozzle will be considered.

2.4 The Design of the Supersonic Convergent-Divergent Nozzle (SCDN):

The method of characteristics was first applied to supersonic flows by Prandtl and Busemann in 1929 and has been much used since. Puckett, in his significant paper in 1946 [6], used this method with supersonic nozzle design and made the technique more accessible to engineers.

In supersonic nozzle design the conventional two-dimensional nozzle is usually considered to consist of several regions as shown in figure (1) These are :-

- (i) the contraction, in which the flow is entirely subsonic,
- (ii) the throat region, in which the flow accelerates from a high subsonic to a low supersonic speed,
- (iii) an initial expansion region, where the slope of the contour increases up to its maximum value,
- (iv) the straightening or 'Buseman-' region in which the cross sectional area increases but the wall slope decreases to zero, and
- (v) the test section, where the flow is uniform and parallel to the axis.

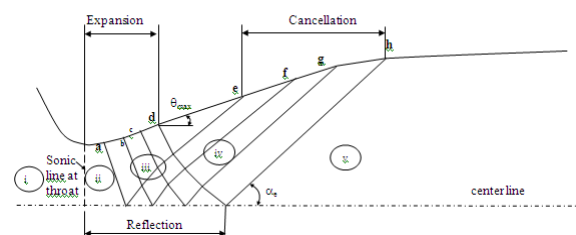


Figure 1. Supersonic nozzle design by the method of characteristics

The equations of motion assume such a form that they may be solved graphically, in a step-by-step process. A two-dimensional flow field in which the velocity is everywhere supersonic can always be represented approximately by a number of small adjacent quadrilateral flow fields in each of which the velocity and pressure are constant. These quadrilaterals must be separated by lines representing waves in the flow; changes in velocity and pressure through any wave can be computed. By increasing the number of small areas into which the complete flow is divided, the accuracy of this approximate solution may be increased without limit. Because of symmetry, the centerline of the nozzle is treated as a solid boundary, and only one half of the nozzle need to be considered for calculation. The gas is to be accelerated to uniform and parallel supersonic flow at the desired Mach number.

The design of the convergent part (region i) is less critical than the divergent part as it contains a subsonic flow [16]. The contraction, from supply section to the throat, must be such a shape that the air is uniformly accelerated and to ensure that the pressure decreases monotonically without any adverse pressure gradient or accrues of separation. This can be achieved by changing the curve smoothly. The length of this section has a little effect on the boundary layer thickness at the throat. The wall curve of this section has been considered as part of a circle with a diameter equal to the throat height started

from the throat area. The length of this section can be any value between 0.5 and 1 of the height of the throat. This value depends on the application and whether long or short nozzle is required. The smallest value in the range will be used, in this work, for the short nozzles and the largest value will be used for the long nozzles.

The divergent part (regions iii-iv) of the nozzle is designed according to the characteristic method using the maximum expansion angle. Saad [17], showed that the gas accelerates beyond the throat (region ii) until the desired Mach number is reached, and the cross-sectional area of the nozzle must increase downstream of the throat. The nozzle contour must then decrease more and more gradually the closer it comes to final area, so that there will be one-dimensional parallel flow at exit (region v). The first portion of the divergent part (region iii) is build in such a way that the contour of the nozzle first turns through a positive angle in the region from point a to point d, and then turn back through the same angle from e to h. Point d thus corresponds to the maximum inclination of the wall (θ_{max}). The contour of the nozzle from a to d is divided into a number of straight segments ab, bc and cd. The expansion of the gas flowing from section a to d is treated as a series of expansion across the waves generated at a, b, c, and d. When those waves reach the centerline, they are reflected, and these reflected waves then intersect other incident waves until they finally reach the nozzle contour at point e, f, g, and h. In the case of the intersection of two waves of opposite families each one will deflect $d\theta$ on its side. Practically, this has been achieved by constructing this portion (region iii) in such a way that the area increases by a finite number of straight line each one millimeter in length and inclined by 2 degree more than the previous one. This process will be continued until the angle of maximum expansion (θ_{max}) is reached. The more points selected in the region between a and d, the larger is the number of the waves that are considered in the analysis and the more effective is the nozzle in providing the desired flow. The nozzle contour a-b-c-d turns the flow through the angle θ_{max} . Its length can be made minimum provided no flow-separation or boundary-layer effects are generated. Any expansion waves that are generated in this portion of the nozzle must be canceled. The nozzle at each impingement point is of such contour that any reflected waves are canceled, and the resulting flow is then parallel and wave-free. The maximum turning angle of the nozzle counter angle θ_{max} occurs at the end of expansion (at point d). At this point the wall will start inclining (region iv) downward by 2 degree at the end of each of reflected Mach wave until it becomes parallel to the axis again as explained previously. Whilst doing this step-by-step graphical procedure, the rules of intersection of two waves and intersection of wave and wall have been used in the way explained above and details can be found in the literatures.

This method was graphical and approximate so that it was subject to protractor errors, and afforded no means of determining the length of the nozzle in advance.

The nozzle design method described above can be used to calculate the 'potential outline'. However, in real fluids, many other factors can affect the design. One of those factors is that it is necessary to allow the growth of the boundary layer along the walls of the tunnel [6]. This is

done by displacing the potential outline away from the tunnel centre line, the correction being applied from knowing the displacement thickness of the boundary layers. Another factor is the shape of throat region. There are two categories for this nozzle according to the sonic line [18-22]. If the sonic line is a straight line, as in this work, the wall at the throat generates centered and divergent waves. The second category has a curved sonic line. In this case, the flow inside the nozzle has no centered Mach lines. This type of nozzle is named Plug nozzle with curved sonic line. One more factor is to include the high temperature effect. This will modify the final shape of the profile as shown by many references [e.g. 21]. The last factor to be mentioned here is the case of the nozzle of the minimum length [21]. This nozzle is characterized by the absence of an initial expansion region. For this case, a series of expansion wave take place at the sharp corner that is called Prandtl-Meyer expansion fan.

3. The Methodology Of The Research

The nozzle design method described above can be achieved in a mechanical manner by using the computer as an aid technique. The problem is solved by developing a software package to perform it. Figure 2 presents the main window of the software package and Figure 3 presents the Calculations and Results window and the details can be found in the Appendix.

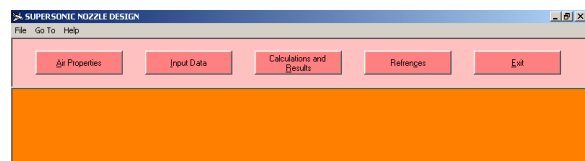


Figure 2. The main window of the package

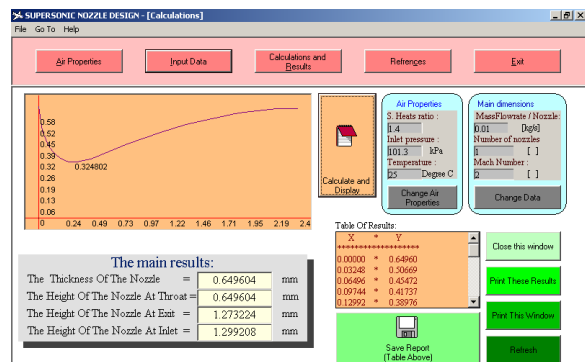


Figure 3. The calculations and results window

Having covered the theoretical background of the design and the features of the software it is appropriate at this point to describe the logic used in this work. The research consists of two stages. The first stage deals with the preparation of databases required whilst the second stage deals with writing the code of the design package that use these data. At the first stage, many graphs have been drawn manually using a standard drafting package, namely, AutoCAD with the aid of Microsoft Excel as a calculation tool. At this stage, graphs for several Mach numbers at a unit throat height have been drawn. These graphs represent the unit model of the nozzles for each value of Mach number. Figure 4 shows the unit models for nine different values of Mach numbers ranging from 1 to

2.5. Each unit model represents a nozzle that gives a specific Mach number with a unit height.

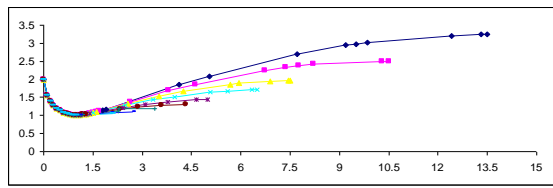


Figure 4. The unit models for nine different values of Mach numbers ranging from 1 to 2.5.

The nozzle may be drawn symmetrically about the center line, or, as is done here, the center line is replaced by a solid wall; the flow solution must be the same in either case[6]. The method of characteristic, described above, has been used step-by-step graphically to construct these models. The x-y coordinates of these graphs have been stored in the unit matrices in the database and then exported into the VB code. At the second stage, a Visual Basic code has been developed to receive the operating conditions from the user and then scaling the unit model to match these conditions.

The main inputs to be entered to the program are the mass flow rate and the required Mach number. The mass flow rate is the main factor to determine the size of the nozzle (i.e. the throat height). One of the menus of the software prompts the user to enter the value of the total mass flow rate required by the application and allow the user to divide this value by the number of nozzles if conditions suggest that. Since the size of the nozzle should not exceed some practicable value, it is suggested here to change the number of the nozzles in order to decrease the flow rate per nozzle and, sequentially, decrease its size. On the other hand, the required Mach number at the exit is the main factor to define the profile of the nozzle and the area ratios. Fortunately, this profile is depending on the geometry only and can be scaled to any size or revised by changing x and y in the same ratio. This is the idea behind extracting infinite number of nozzles from the few unit models explained above. This can be done by finding the ratio between the required throat area and the throat area of the appropriate unit model. This ratio is the scale factor that converts the unit model into the profile of the required nozzle.

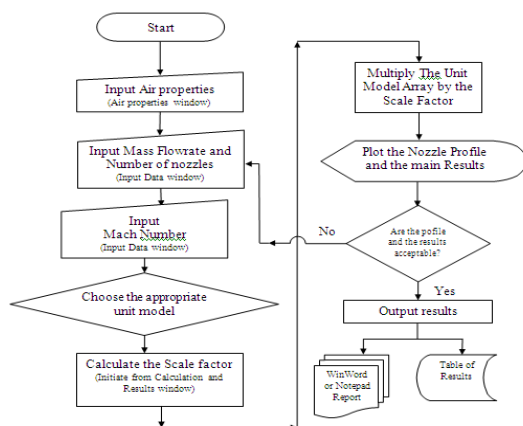


Figure 5. Presents the flowchart of the main program.

The following is a pseudo code presentation of the algorithm of the main program:

Algorithm:

- Step 1: Choose the working fluid and provide its properties.
- Step 2: Enter the total mass flow rate and the number of nozzles.
- Step 3: Enter the required Mach number at the nozzle exit.
- Step 4: Initialize the calculation and evaluate the scale factor by comparing the area required by the mass flow rate and the area of the unit model.
- Step 5: Choose the appropriate unit model among the available database according to the required Mach number at the nozzle exit.
- Step 6: Multiply the array of the unit model by the scale factor and this will create the final array.
- Step 7: Plot the x-y profile of nozzle from the final array.
- Step 8: Change input data if required.
- Step 9: Send the final array into the list box.
- Step 10: Send the final array into a file if a soft copy is required.
- Step 11: Send the whole screen into a printer if a hard copy is required.
- Step 12: Terminate the session.

4. Results and Discussion

In this section some examples of how to use the program and few study cases have been presented. Figure 6 presents an example of how to use the program. This figure has been extracted from the program for the same Mach number (2.3688) and for five different values of flow rate. The figure shows the similarity of the five nozzles and the only difference is the size that changes by the scale factor.

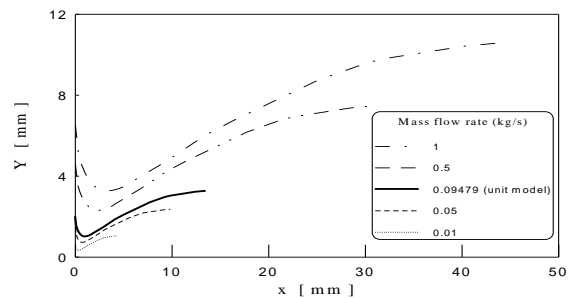


Figure 6. Comparison of a nozzle profile for five different values of flow rate and the same value of Exit Mach No. of 2.3688.

The functionality of the software has been validated by using it to solve many problems with known solutions. The software displayed logical results (study cases are shown in figures 7-9). Figures 7 and 8 show a very high degree of agreement between the manual methods [3,6] and computerized method of this work.

The diversity shown in figure 9 between the package results and the result of Puckett [6] is due to the approximations used in the latter work, and as expected the nozzle become longer. The approximation was necessary before the use of computer; however, the new results look more accurate.

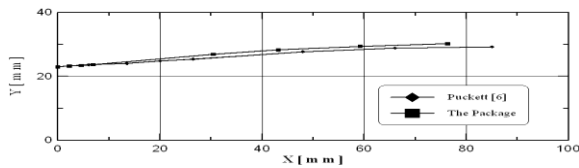


Figure 7. Case1: Comparison of a nozzle profile of Exit Mach No. of 1.64 between the values extracted from the Package and from Fig. 8 of [6].

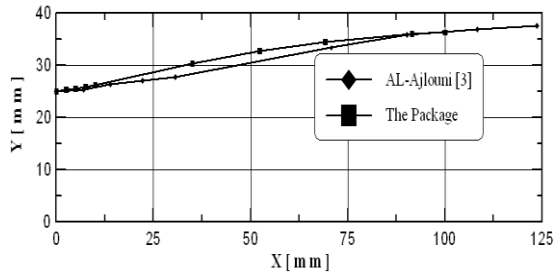


Figure 8. Case2: Comparison of a nozzle profile of Exit Mach No. of 1.77 between the values extracted from the Package and from Figure (7.2) of [3].

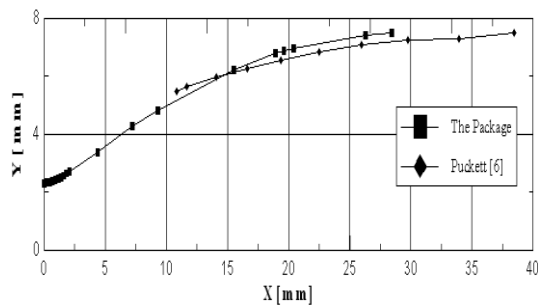


Figure 9. Case3: Comparison of a nozzle profile of Exit Mach No. of 2.42 between the values extracted from the Package and Table 2 of [6].

5. Conclusions

Supersonic convergent-divergent nozzles (SCDN) have many applications. They are usually subjected to complex flow pattern. Different degrees of simplification have been studied. Computer is a must to achieve high accuracy and large calculation needed with modern high speed applications. Hence, a computerized approximation approach might be a better method to tackle such a problem. Characteristic method is the most appropriate method to be used with (SCDN) design. Programming of this method directly is impractical as it uses a step-by-step graphical procedure. This problem has been overcome by using the unit models as a database for the developed package. The unit models have been built by using the method of characteristic for a representative number of nozzles that cover the required range of Mach numbers. These models have been established as the advocate database for the software package under development.

This research presents the development of a software package that can be considered as a tool for designing a (SCDN). The software is very user friendly and provides a wide range of options such as filing, plotting, and reporting. The profile of the nozzle in both graphical and tabulated form is the main output. The developed software

includes also an error tracking system. The functionality of the software has been validated by using it to solve many problems with known solutions. The software displayed logical results

It shows that the developed software is a very useful tool for the designer in producing an accurate and immediate solution for very complicated cases. Any way, this is not the final version of the software and many other facilities could be added. For example, more general conditions may be considered. The future version of this software may include extension for higher range of Mach number, calculation of the mass flow rate, boundary layer correction, High temperature compensation, Minimum length nozzle, other types of the nozzle cross section, and a flexible link with other CAD packages.

Encouraging results are obtained so far. However, more future research must be made to develop a more sophisticated and flexible version of the package

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Appendix A. Appendix Description of the software package:

The developed software package was implemented using the programming language Visual Basic (VB6). It consists of many standard forms like the splash screen, the log in dialog (password), the about dialog, the common dialogs (like open, save and color), introductory menu, and others. Figure 2 presents the main window of the software package. This window is one of the important interface windows throughout the software package. The type of this window is a multiple document interface (MDI) and it is a parent window for the other windows. Through this window the designer can activate the following buttons that open the child windows

1. Air Properties.: to open the window to be used to enter the working fluid properties: e.g., the specific heats ratio, inlet pressure, and the inlet temperature.
2. Input data: to open the window to be used to enter the mass flow rate, the number of the nozzle and the required Mach number.
3. Calculations and Results: to open the window to initialize the calculations and to choose the type of the output. This is the most interactive window and will be described afterward in more detail.
4. References: to link with WinWord document that contains details about the references.
5. Exit: to terminate the package when needed.

Moreover, other buttons and menu items can be activated from this window. Figure 3 presents the Calculations and Results window. On this window, the user can start the calculation after the inspection of the inputs. The nozzle profile will be displayed as a plot using the most advanced plotting commands to add a dynamic axis, scale and a tracing point. The same information can be extracted in a listed form through a dynamic list box. This list box has been linked internally to the plot area. Clicking any value in the list box will be displayed directly in the plotting area as the tracing point. Other type of output can be obtained as a soft or hard copy of the results by importing data from the software into a document file (WinWord), text file (WordPad) or printed hard copy. In addition, the software package includes an error tracking system of messages. For example, it reminds the user of missing data, or illogical entries.