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Structural Reliability of Thin Plates with Random Geometrical Imperfections Subjected to Uniform Axial Compression

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Abstract

It is generally known that the load carrying capacity of thin plate structures under axial compression are partially dependent on the initial geometric imperfections. Since these initial geometric imperfections are random in nature, the strength distribution will also be random. Hence a probabilistic approach is required for a reliable design of these thin plate structures. In this paper, by keeping the variance of imperfections of all the models at assumed manufacturing tolerance of 1.71 mm and maintaining the maximum amplitude of imperfections within ±8 mm, 1024 random geometrical imperfect plate models are generated by the linear combination of first 10 eigen affine mode shapes using 2^k factorial design. These imperfection models are analyzed using ANSYS non-linear FE buckling analysis including both geometrical and material nonlinearities. From these FE analysis results, the strength distribution of the plate is obtained and using Mean Value First Order Second Moment (MVFOSM) method, reliability analysis is carried out.

Keywords: Buckling Strength; Thin Plates; Geometrical Imperfections; Random Modeling; Reliability Based Design.

Nomenclature

E	Young's modulus in N/m^2
$f_G(g)$	Distribution of failure function
G	Failure function
i	Number of nodes
j	Number of eigen affine mode shapes.
l	Length of the plate in m
L	Load distribution
m	Number of transverse half lobes
M	Modal imperfection magnitude vector
n	Number of longitudinal half lobes
P_f	Probability of failure
R	Reliability of structure
ROTZ	Rotationabout z-direction
S	Strength distribution
t	Thickness of the plate in mm
Ux	Displacement along x-direction
Uy	Displacement along y-direction
Uz	Displacement along z-direction
w	Width of the plate in m

Greek Letters

γ	Poisson's ratio
σ_y	Yield strength in MPa
ρ	Density in kg/m^3
Δ	Nodal imperfection amplitude vectors in m
σ^2_Δ	Variance of Nodal imperfection amplitude vectors
σ^2_M	Variance of Modal imperfection amplitude vectors
σ^2_{tol}	Variance of tolerance
μ_Δ	Mean of Nodal imperfection amplitude vectors
μ_M	Mean of Modal imperfection amplitude vectors
Φ	Eigen vector matrix
Φ^*	Pseudo inverse of Φ matrix
Φ^T	Transpose of Φ matrix
μ_S	Mean of strength distribution
μ_L	Mean of load distribution
σ_S	Standard deviation of strength distribution
σ_L	Standard deviation of load distribution
φ	Cumulative normal distribution function
β	Safety index

Abbreviations

BSR	Buckling Strength Ratio
MVFOSM	Mean Value First Order Second Moment Method
RSM	Response Surface Methodology

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1. Introduction

Thin plate structures are widely used for applications like mechanical, marine, aerospace and in civil engineering structures. The manufacturing process involved in making perfect thin plate is difficult because there will be some geometrical imperfections like local indentations, swelling, non-uniform thickness etc., and material imperfections like heterogeneities, cracks, vacancies, impurities etc., and also other imperfections like residual stresses and strains induced during manufacturing. These imperfections generally affect the buckling behavior of plates such that studying about this effect requires complete information about their nature. Among these imperfections, geometrical imperfections are more dominant in determining the load carrying capacity of thin shell structures. Reliable prediction of buckling strength of these structures is important because buckling failure is catastrophic in nature. The geometrical imperfections present in these structures are highly random in nature which requires probabilistic approach to determine the safe load for these structures.

The Joint Committee on Structural Safety classified the structural reliability design analysis and the safety checking into three groups namely level 1, level 2, and level 3 methods [1]. In level 1 method, appropriate levels of structural reliability are provided on a structural element basis by specifying a number of partial safety factors, related to some predefined characteristic values of the basic variables. In level 2 method, safety checks are done only at selected points on the boundary as defined by appropriate limit stage equations in the space of the basic variables using the mean and variance of these variables. In level 3 method, safety checking is done based on exact probabilistic analysis for whole structure systems or elements using a full distribution approach based on failure probabilities, possibly derived from optimization studies or assessed by other approaches.

Level 2 methods are more practical oriented and quite suitable for design, and they are also used for calibration of codes for the evaluation of partial safety factors in a rational manner. Hence in this paper, the level 2 Mean Value First Order Second Moment (MVFOSM) Method is adopted to determine the safe critical load of the plate.

2. Literature Review

The modeling of imperfections can be classified into deterministic and random geometrical imperfection modeling. In case of the deterministic approach, imperfections are either obtained from actual measurement (for example, Athiannan and Palaninathan [2], Elishakoff and Arbocz [3], Arbocz and Hol [4], Scheneider [5], Singer [6]) or from assumed imperfection pattern. The assumed imperfection pattern may be axisymmetric (for example, Pircher et al [7] and Khamlichi et al. [8]) or first eigen mode shape pattern (for example Teng and Song [9], Kim and Kim [10], Khelil [11], Featherston [12]), or a combination of axisymmetric and asymmetric (for example, Arbocz and Babcock [13] & Arbocz and Sechler [14]) mode shape pattern.

There are two ways by which random modeling of imperfections can be achieved. The first is by varying the nodal locations of the structural model randomly, and the second is the stochastic FE approach.

Each manufacturing process has its own characteristic shapes that can be represented by double Fourier series. In the earlier studies, these Fourier coefficients were made as random variables to get different random geometrical imperfection models (for example Athiannan and Palaninathan [2], Scheneider [5]).

Amazigo and Budiansky [15] studied buckling strength of cylindrical shells with random axisymmetric type geometrical imperfections pattern and derived asymptotic formula for maximum load carrying capacity of the cylindrical shell. Elishakoff [16] gave a reliability method based on Monte Carlo simulation technique, and applied that to the problem of buckling of finite column with initial geometrical imperfections, which is assumed as Gaussian random fields.

In the work of Elishakoff and Arbocz [3], the random geometrical imperfections were represented by the buckling modes of its perfect structure in Fourier form. By varying the Fourier coefficients at random, large numbers of imperfect cylindrical shell models were created and their buckling strengths were determined. Elishakoff et al. [17] explained about the MVFOSM method to predict the reliability of cylindrical shell possessing axisymmetric and asymmetric random geometrical imperfections. For this, they used the second order statistical properties obtained from measured initial geometric imperfections. Results of reliability calculations were verified with results from Monte Carlo simulation.

Chryssanthopoulos et al. [18] presented Response Surface Methodology (RSM) to determine the reliability of stiffened cylindrical and plate shells subjected to axial compression, considering the manufacturing variabilities such as initial geometrical imperfections and welding residual stresses.

In the work by Arbocz and Hol [4], using MVFOSM method, the reliability of the isotropic, orthotropic and anisotropic circular cylindrical shells under axial compression were evaluated, assuming Fourier coefficients obtained from measurement of geometrical imperfections as random. Chryssanthopoulos and Poggi [19] generated random geometrical imperfections by varied Fourier coefficients multimode combinations of characteristic wave forms obtained from measured random geometrical imperfections. A suggestion was given that in the absence of detailed statistical analysis and characteristics shapes, it is reasonable to estimate the knockdown factor of the cylindrical shells by setting the maximum amplitude of an imperfection mode (which is affine to buckling mode or as a combination of two modes) equal to average maximum imperfection measured on the specimen.

Warren [20] generated random geometrical imperfections by linear combinations of eigen buckling affine mode shapes using 2^k factorial design of Design of Experiments (DoE), and the variance of the models were maintained within the tolerance of manufacturing and adopted RSM to determine reliability of framed structures. Náprstek [21] explained about stochastic finite element methodology taking large displacement as source of nonlinearities and studied about the response of the structures with random imperfection of Gaussian type.

Bielewicz et al. [22] developed a simulation method to generate random geometrical imperfections using non-homogeneous two dimensional random fields on regular nets. Schenk and Schueller [23] in their work, using imperfection databank at Delft University of Technology, generated geometrical imperfection models utilizing Karhunen-Loéve expansion method. From the deterministic analysis of random models, buckling strength distribution was obtained from which the reliability of the structure is determined using Monte Carlo Technique.

Papadopoulos and Papadrakakis [24] developed a nonlinear triangular composite element to carry out stochastic structural stability analysis of thin shell structures with random geometrical initial imperfections, which can be described as a two-dimensional univaried homogeneous stochastic field. Craig and Roux [25] also used the Karhunen–Lo`eve expansion as a method to incorporate random geometrical imperfections into the FE buckling analysis and verified the numerical results with other numerical and experimental results.

In the present work, random geometrical imperfections are generated using first 10 eigen affine mode shapes of perfect plate taken for study as suggested by Arbocz and Hol [4], Chryssanthopoulos and Poggi [19] and combine linearly, following 2^k factorial design of Design of Experiments (DoE) and the variance of the models were maintained within the tolerance of manufacturing as suggested by Warren [20]. From the deterministic FE analysis, strength distribution is obtained from which the reliability of structure is determined using MVFOSM method..

3. Finite Element Modelling

An eight noded quadrilateral shell element, SHELL93 of ANSYS is used for modeling the thin plates. This element can handle membrane, bending and transverse shear effect, and also able to form curvilinear surface satisfactorily. This element also has plasticity, stress stiffening, large deflection and large strain capabilities.

3.1. Thin Plate Shell Model

The thin structural steel plate model taken for study is [26]:

Length (l) = 1m
Width (w) = 1m
Thickness (t) = 8 mm
Poisson's ratio (γ) = 0.3
Young's modulus (E) = 205.8 GPa
Yield stress (σ_y) = 313.6 Mpa
Density (ρ) = 7800 kg/m^3
Zero strain hardening effect is assumed.

3.2. Boundary Conditions

Simply supported boundary conditions as shown in Figure 1, are applied on all the edges of the thin plate and uniform displacement loading is applied on one side of the plate model and corresponding opposite side is restrained from moving along load direction [26].

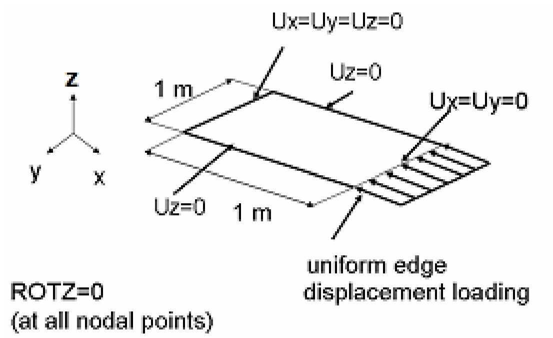


Figure 1. Geometry, boundary, and loading conditions used in buckling analysis of a thin plate (not to scale)

3.3. Model Validation and Determination of Eigen Affine Mode Shapes

3.3.1. Eigen Buckling Analysis

The mesh convergence study is done to choose the optimum number of elements for the analysis, and it is found that 40 elements along both directions give accurate solution, and hence same number of elements is used for all analysis. The analytical solution [27] of the perfect thin plate is compared with the FE eigen buckling analysis result and is shown in Table 1 such that FE model validation is ensured. In the table, m and n represent the number of transverse and longitudinal lobes respectively.

Table 1. Comparison of analytical solution with FE eigen buckling analysis result.

Mode.No	m	n	Analytical Solution(N)	FE Solution(N)	% Error
1	1	1	380936	378115.3	0.7404
2	2	1	595213	591989.6	0.5415
3	3	1	1058160	1053573.8	0.4334
4	2	2	1523750	1510774.2	0.8515
5	4	1	1720170	1712254.2	0.4601
6	3	2	1788280	1773286.6	0.8384
7	4	2	2385590	2361591.1	1.0059
8	1	2	2385590	2367998.9	0.7373
9	5	1	2580260	2560567.4	0.7632
10	5	2	3210060	3176441.5	1.0472

3.3.2. Non linear FE analysis

Validation of the developed FE model is done with the published result of Suneel Kumar et al. [28]. For this purpose, an unstiffened plate of dimensions 500x500x3.2 mm is taken for the study with the following material properties: Yield strength = 264.6 MPa, Young's modulus = 205.8 GPa and Poisson's ratio = 0.3. For a mesh size of 20x20 elements, the ultimate strength of the plate obtained from nonlinear FE analysis is 394.96 kN. The ultimate strength of the same plate, given by Suneel Kumar et al., is 392.93 kN [28] which is very much in agreement with the result obtained from nonlinear FE analysis.

3.4. Modeling of Imperfect Plates

To ensure the amplitude of imperfections at any nodal point of FE model (except the nodes at boundary edges) to be random, the first ten eigen affine mode shapes of linear buckling of thin plate are combined linearly using 2^k factorial design of Design of Experiments (DoE).

The modeling of the initial random geometrical imperfections is accomplished using the following assumptions/ conditions.

- Δ - imperfection amplitudes at all nodes except the nodes at the boundary edges should follow independent normal distribution.
- Mean value of imperfection amplitude of a node from all random models should be made equal to zero.
- Equal importance should be given for the all eigen affine mode shapes considered for random modeling.
- The random imperfection shapes generated should be linear combinations of the eigen affine mode shapes considered.

Based on the above assumptions, the nodal amplitude of imperfection vector for the entire structure (except the edge nodes, where the displacements are constrained) is given by

$$\Delta_{i\times 1} = \phi_{i\times j} \times M_{j\times 1} \quad (1)$$

where, Δ- Nodal imperfection amplitude vector

ϕ- The matrix of eigen vectors containing the modal imperfection amplitudes at all nodal points of selected eigen affine mode shapes with equal maximum amplitude of imperfections

M- Modal imperfection magnitude vector

i- number of nodes

j- number of eigen affine mode shapes

If the nodal amplitudes of imperfections are known, the modal imperfection magnitudes can be obtained using the relation

$$M_{j\times 1} = \phi^{*}_{j\times i} \times \Delta_{i\times 1} \quad (2)$$

where the matrix ϕ^* is the pseudo-inverse of the matrix ϕ. The pseudo-inverse is calculated using the following equation based on method of least squares

$$\phi^{*} = \left(\phi^{T}\phi\right)^{-1}\phi^{T} \quad (3)$$

If the nodal imperfections Δ_i are independent and normally distributed random variables then the mean value and variance of each modal magnitude is given by

$$\mu_{M\,j} = \sum_{1}^{j} \phi^{*}_{ji}\, \mu_{\Delta i} \quad (4)$$

$$\sigma^2_{M\,j} = \sum_{1}^{j} \left(\phi^{*}_{ji}\right)^2 \sigma^2_{\Delta i} \quad (5)$$

where, $\mu_{\Delta i}$ and σ^2_{Δ} - mean and variance of the nodal imperfection amplitude.

μ_M ,σ^2_M - mean and variance of the modal imperfection magnitude.

Similarly, mean and variance of each nodal amplitude is given by

$$\mu_{\Delta i} = \sum_{1}^{j} \phi_{ij}\, \mu_{M\,j} \quad (6)$$

$$\sigma^2_{\Delta i} = \sum_{1}^{j} \left(\phi_{ij}\right)^2 \sigma^2_{M\,j} \quad (7)$$

Since it is required to have nodal amplitude Δ_i of any node i of the structure to follow normal distribution with μ_Δ=0 and as per Eq. 4, μ_M also becomes zero. Hence, to get amplitude of imperfections of all nodes for each model, the modal magnitude of each model has to be obtained by using Eq. 5. Using the modal magnitudes obtained from previous step, the nodal amplitudes of imperfections can be obtained by using the Eq. 1. Thus by varying the modal magnitudes of imperfections randomly using 2^k factorial design matrix of Design of Experiments, random geometrical imperfection models can be generated.

3.5. Steps Followed in Random Geometrical Imperfections Modeling

Step –I

Initially, substitute variance of modal imperfection magnitude vector as

$$\sigma^2_M = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (8)$$

STEP-II

Using Eq.(7) the variance of nodal imperfection amplitude vector σ^2_Δ is determined.

Step –III

Each element of the resulting σ^2_Δ vector from Step-II is normalized with the maximum value of element in that vector. Each value of normalized σ^2_Δ is multiplied by σ^2_{tol} so as to limit the maximum amplitude of imperfections.

Step –IV

Using the σ^2_Δ vector obtained from the Step-III, new σ^2_M vector is found using Eq.5.

Step –V

Since, μ_Δ = 0, μ_M = 0, using σ^2_M new vector determines the modal imperfection magnitude vector M such that $M = \pm\sigma_M$.

Step –VI

Using 2^k factorial design, design matrix is generated, and each column of design matrix is selected and is multiplied by corresponding element in the M vector obtained from previous step. This new design matrix is used to generate 2^k (for k=10, 2^{10} = 1024) random geometrical imperfection models.

$$\Delta = \phi \text{ x new design matrix} \quad (9)$$

With the value of modal imperfection magnitude vector M, Δ nodal imperfection vector is determined using the Eq.1. But the ± value of the modal imperfection magnitude is decided by +1 or -1 of the design matrix obtained from DoE. The Δ matrix, thus formed has 1024 rows, and each row corresponds to nodal displacements of all nodes of one random imperfect plate model. By adopting the procedure explained above, 512 pairs of mirror image random imperfect plate models can be generated.

Here, in this work, 1024 random geometrical imperfect plate models are generated keeping RMS value of imperfections = 1.711 mm, and the maximum amplitude of imperfection is maintained within ±8mm. The maximum amplitude of imperfections in all 1024 models is shown in Figure 2. From this Figure it can be noted that maximum amplitude of imperfections from model number 1 to 512 are exactly mirrored between model numbers 1024 to 513.

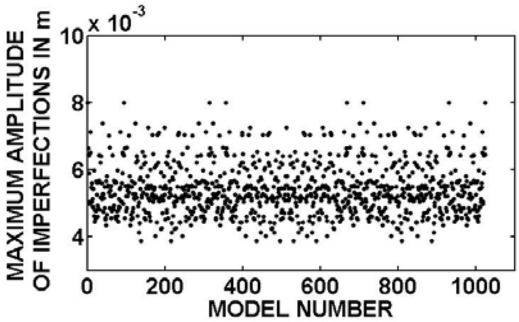


Figure 2. Scatter of maximum amplitude of imperfections from 1024 models

A sample of a pair of thin plate models with mirror image random imperfections are shown in Figure 3.

To verify the assumptions made that imperfection amplitude of a node except boundary nodes are randomly distributed**,** the distribution of out of plane displacements of a particular node from all 1024 random plate models is plotted as shown in Figure 4 (a) and (b).

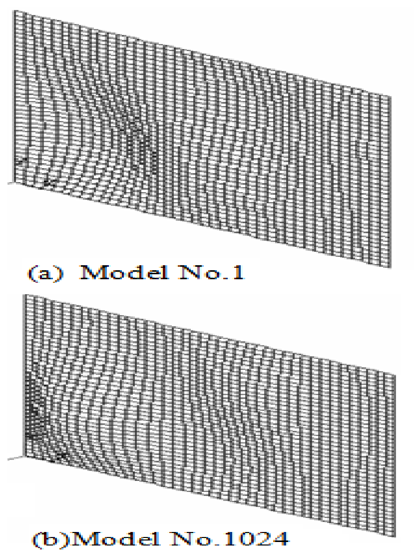


Figure 3. A pair of mirror image random imperfections plate models (amplitude enlarged by 50 times)

From Figure 4, it can be seen that the out of plane displacement of nodal point distribution follows normal distribution with mean $(\mu_\Delta) = 0$.

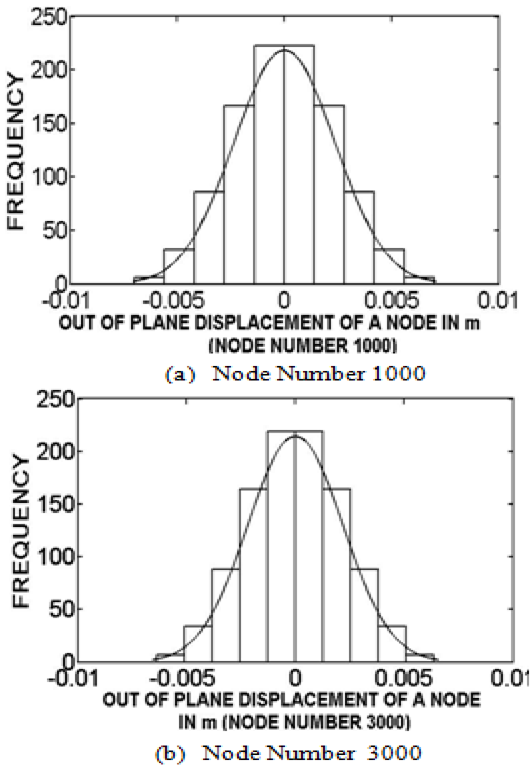


Figure 4. Normal distribution of out of plane displacements of a particular node from all 1024 random plate models

4. Reliability Analysis

For any structure, the strength and load are highly probabilistic, assuming that the strength (S) and load (L) are normally distributed as shown in Figure 5.

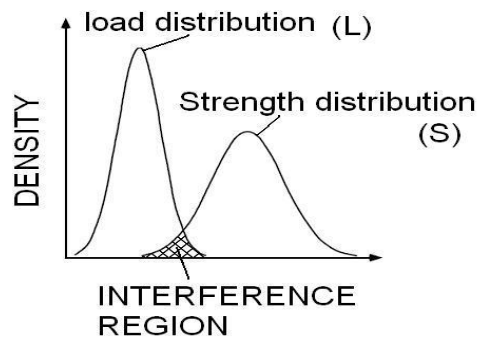


Figure 5. Load and strength distributions.

Let us define the failure function as,

$$G = S - L \quad (10)$$

Then, the distribution of failure function f_G (g) is shown in Figure 6. The probability of failure of the structure is given by

$$Pf = P(G < 0) \quad (11)$$

The reliability of the structure is given as,

$$R = 1 - Pf \quad (12)$$

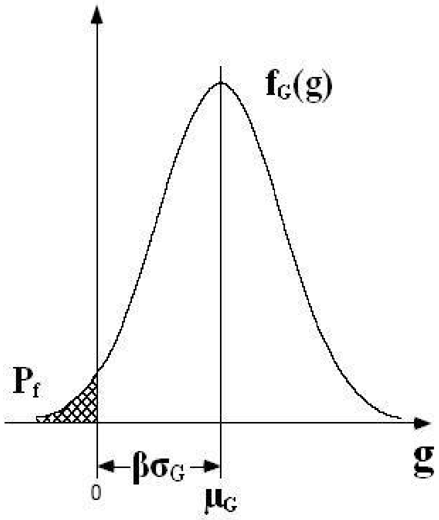


Figure 6. Normal distribution of failure function

In the MVFOSM method of determining the reliability of the structure, the mean and variance of the random variables (in this case, the strength and load) are considered.

The first order approximation of failure function f_G (g) is used for finding the mean and variance of the failure function. Thus, the mean and variance of the strength and load variables are required in order to carry out the reliability analysis.

5. Results and Discussion

Using nonlinear FE analysis, buckling strength of first 512 models is determined including both material and geometrical nonlinearities. Determining the buckling strength of the next 512 models is nothing but inverting the first 512 models and obtaining the buckling strength. For reliability calculation buckling strength of 1024 models or first 512 models can be considered because it will not affect the reliability calculations. By considering 1024 models, only the frequency of buckling strength values occurrence will be doubled. But here for reliability calculation, buckling strength ratio (BSR) values of first 512 models are considered, where BSR can be defined as ratio between buckling strength of imperfect plate to that of the first eigen buckling strength of perfect plate. Since thin plates are having positive post buckling behavior, its BSR values are greater than 1 [12]. Figure 7 shows the BSR values of first 512 random models.

Figure 8 shows the stiffness curve obtained for model number 1. From this Figure, it can be seen that at limit load condition, the plate structure fails as the slope of the stiffness curve becomes zero. Figure 9 shows the von Mises stress contour obtained for model 1 at limit load condition.

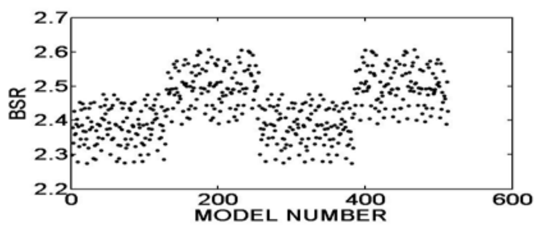


Figure 7. Model Number Vs BSR

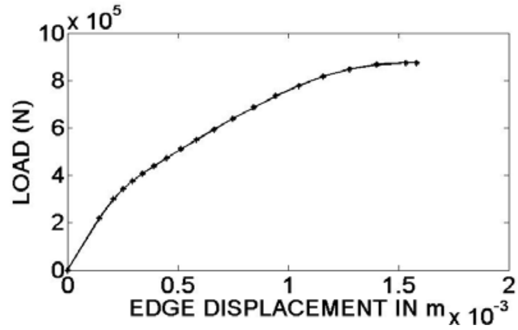


Figure 8. Stiffness curves random imperfection of model 1

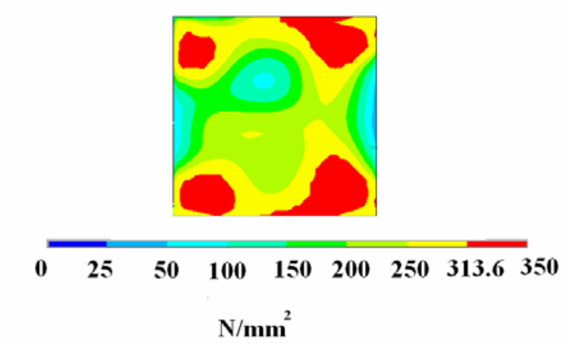
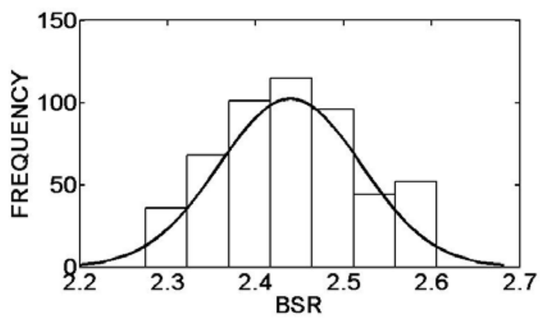


Figure 9. The von Mises stress contour of random imperfection of model 1 at its limit load condition.

To obtain the strength distribution for reliability analysis, the BSR values of 512 random models are considered and are shown in Figure.10. From the figure, it can be seen that, the distribution does not follow normal distribution exactly but it is a skewed normal distribution.

Since the normal distribution shape is the simplest, best developed, most known and expedient [29], the skewed strength distribution is converted into an equivalent normal distribution using Central limit theorem.



Mean of distribution = 2.4393
Mode of distribution = 2.4285
Standard deviation = 0.0808

Figure 10. Actual Strength distribution obtained using BSR values

According to Central limit theorem if a random sample of n observation is selected from any population, then, when the sample size is sufficiently large (n>=30) the sampling distribution of the mean tends to approximate the normal distribution. The larger the sample size is, the better the normal approximation to the sampling distribution of the mean will be.

Hence, 100 samples were taken with each set containing 200 observations drawn randomly from BSR of 512 models. The mean of 200 observations, taken randomly in each sample, was calculated, and the means of all 100 samples are plotted in Figure 11. The equivalent normal strength obtained from means of 100 samples is found to satisfy the normal distribution with 5% level of significance as shown in Figure 12.

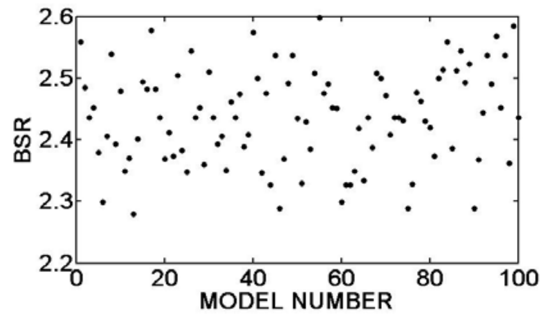
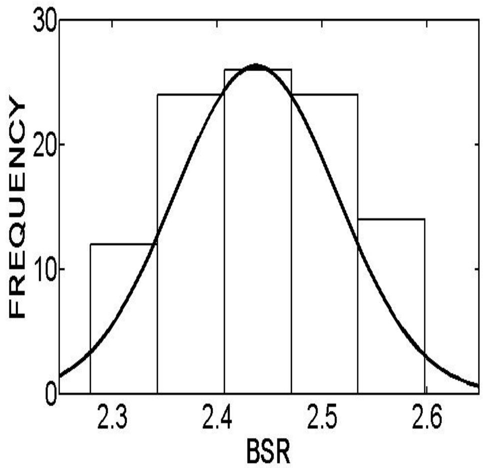


Figure 11. Sample set number and their corresponding mean



Mean of distribution = 2.4369

Mode of distribution = 2.5373

Standard deviation = 0.0775

Figure 12. Equivalent strength distribution obtained by Central Limit Theorem for reliability calculations

The mean of the distribution obtained from Central Limit Theorem and the mean of the actual distribution differ by only - 0.098 %. Moreover, the skewness of the distribution is approximately = 0 (i.e., - 0.05), which also confirms that the distribution is Gaussian.

According to the Mean Value First Order Second Moment (MVFOSM) method, the reliability index is defined as

$$\beta = \frac{\mu_S - \mu_L}{\sqrt{\sigma^2_S + \sigma^2_L}} \quad (13)$$

μ_S = Mean of strength distribution

μ_L = Mean of load distribution

σ_S = Standard deviation of strength Distribution

σ_L = Standard deviation of load distribution

β = Safety index

The probability of failure is given by,

$$P_f = \varphi(-\beta) \quad (14)$$

where φ is cumulative normal distribution function

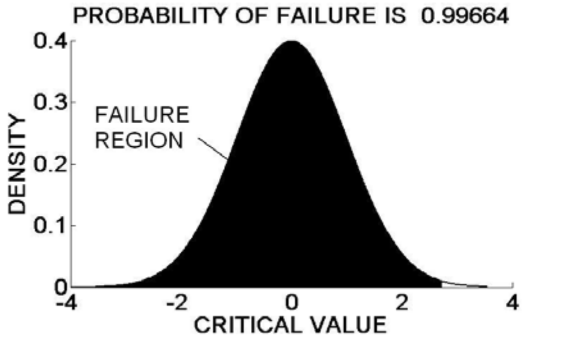
Then, reliability of the structure is given as,

$$R = 1 - P_f \quad (15)$$

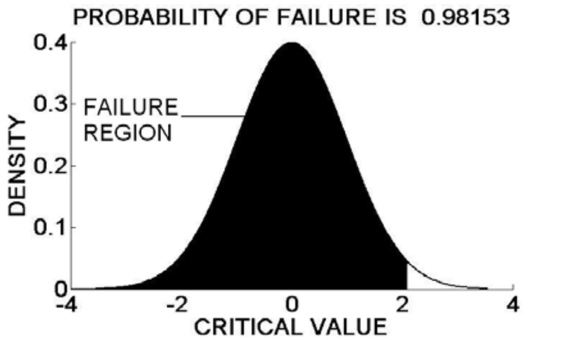
In this case, the load applied is assumed as a deterministic single value. Hence, $\sigma_L = 0$ and now β is defined as,

$$\beta = \frac{\mu_S - \text{Load applied (BSR)}}{\sigma_S} \quad (16)$$

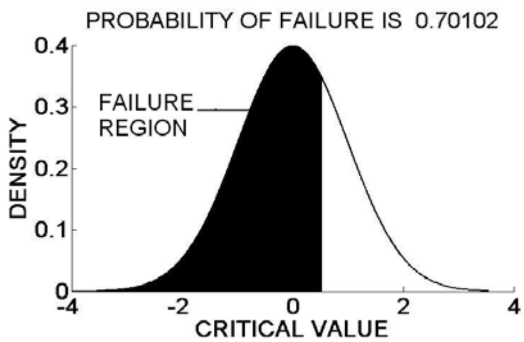
By varying the load applied, the reliability of the structure at each load is obtained. The failure probability at different loads is shown in Figure 13.



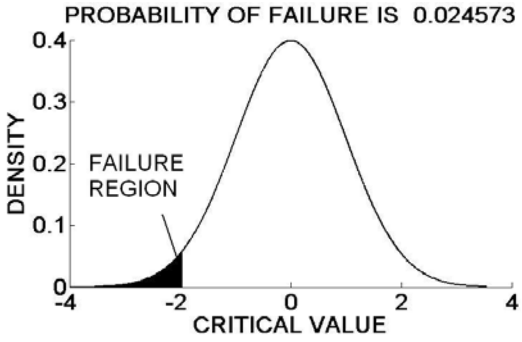
(a) Load applied is 2.602 times more than the first eigen mode buckling strength



(b) Load applied is 2.532 times more than the first eigen mode buckling strength



(c) Load applied is 2.412 times more than the first eigen mode buckling strength



(d) Load applied is 2.25 times more than the first eigen mode buckling strength

Figure 13. Failure probability at different loads.

The variation of reliability with respect to the BSR is shown in Figure 14. From the reliability curve, it can be noted that for the plate taken for study, if BSR is upto 2.1 times of that of the perfect plate, the reliability is100%, and when the load applied is more than 2.65 times the strength of perfect plate, the reliability is zero.

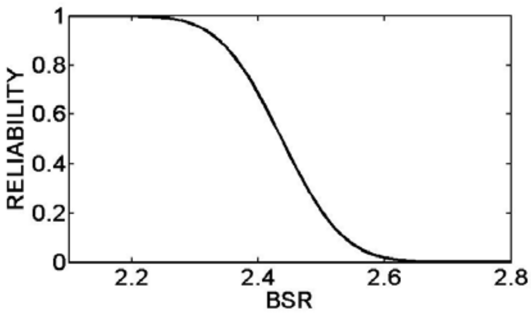


Figure 14. Reliability Vs BSR

6. Conclusions

The following conclusions are derived from the analysis carried out for the thin plate structure taken for study.

- The slope of stiffness curve decreases gradually as the load applied increases and becomes zero at limit load condition and thereby imperfect thin plates collapse.
- To increase reliable prediction of safe load of the structure further, more number of eigen affine mode shapes can be considered.
- Buckling strength of imperfect thin plate is more than two times that of perfect thin plate.
- Using the adopted MVFOSM method of reliability, it is found that the reliability of thin plate taken for study under axial compression is 100% up to 2.25 times of the strength of perfect plate. And the reliability becomes zero when the load applied is more than 2.65 times of the strength of perfect plate.

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