JJMIE
Volume 4, Number 2, March. 2010
ISSN 1995-6665
Pages 314 - 323
Jordan Journal of Mechanical and Industrial Engineering

An Effective Approach for Solving The Multi-Response Problem in Taguchi Method

Abbas Al-Refaie [a,*], Tai-Hsi Wu [b], Ming-Hsien Li [c]

[a] Dept. of Industrial Engineering, University of Jordan, Amman, Jordan

[b] Dept. of Industrial Engineering and Systems Management, Feng Chia University, Taichung, Taiwan

[c] Dept. of Business Administration, National Taipei University, Taipei, Taiwan

Abstract

This research proposes a simple, yet very effective, approach for solving the multi-response problem in the Taguchi method. Each quality response is transformed into signal-to-noise (S/N) ratio. The average S/N ratio is calculated for each factor level, and then weighted with respect to the level of the largest average S/N ratio for this factor. The average weight of each factor level, or level weight, is obtained from all responses. The factor level with the largest level weight is selected as the optimal level for that factor. Three case studies in manufacturing are employed for illustration of the proposed approach; where in all of which provides it the largest total anticipated improvements. In contrast with other approaches in previous literature, the proposed approach not only reduces the complexity and effort of data analysis and the need for statistical skills, but also does not require estimation of a weight for each response, and thus eliminates human involvement in the decision making process for optimal factor levels. In conclusion, the simplicity and effectiveness of this approach shall make it attractive to practitioners for solving the multi-response problem in a wide range of applications on the Taguchi method.

Keywords: Multi-Response Problem; Taguchi Method; S/N Ratio.

1. Introduction

Robust design in quality engineering, as proposed by Taguchi [1], has significantly improved quality and yield in product and manufacturing process design. Nevertheless, most of the published researches [2 and 3] on the applications of the Taguchi method have focused on optimizing a single quality response.

In today's high-tech processes, however, manufactured products have more than one quality response of main interest. The Taguchi method primarily uses engineering judgment to decide optimal factor levels for multi-responses [4], which increases uncertainty during the decision-making process. Recently, the optimization of multi-response problem has received an increasing attention from many authors. Among them, Shiau [5] assigned a weight to the S/N ratio of each quality response. Then, the combined S/N ratio was employed to determine the optimal factor levels. Tong et al. [6] adopted the sum of the weighted normalized quality losses of all responses, then used multi-response S/N ratio to decide optimal factor levels. In reality, it remains difficult to determine and define a weight for each response. Logothetis and Haigh [7] and Pignatello [8] determined tentative optimal factor levels by using regression techniques which increase the complexity of computational processes. Su and Tong [9] and Antony [10] utilized principal component analysis (PCA) to transform multi-responses into few uncorrelated responses, which were then utilized for solving the multi-response problem. However, PCA has two shortcomings: (1) when multiple principal components of an eigenvalue that is greater than one are chosen, how to trade off to decide the feasible solution is still unknown; and (2) when the selected principal components have less variation than can be explained by total variation, the performance index of multi-responses is not evident enough to replace the original response variables. Liao and Chen [11] utilized data envelopment analysis (DEA) based ranking approach (DEAR) for deciding optimal factor levels for multi-response problem in Taguchi method. Unfortunately, most of the traditional DEA models allow for complete weight

* Corresponding author.abbas.alrefai@ju.edu.jo

flexibility which may result in identifying a decision making unit with an unrealistic weighing scheme [12 and 13]. Jeyapaul et al. [14] utilized genetic algorithm (GA) with the Taguchi method to determine a weight for the S/N ratio of each response. The sum of the weighted S/N ratios was then used to decide optimal factor levels. Genetic algorithm, however, is a search heuristic that provides near-optimal solutions for complex search spaces; such as, production scheduling and transportation problems.

This research proposes a simple yet effective approach for solving the multi-response problem in the Taguchi method. The basic idea is that the average S/N ratio for each factor level from each response is weighted with respect to the largest average S/N ratio for that factor. Then, the average weight from all responses is used as a quality measure to decide optimal factor levels. That is, the factor level corresponding to the largest average weight among all factor levels is calculated to be the optimal level for that factor. The remainder of this research is organized as follows. Section two presents the proposed approach. Section three provides three case studies for illustrational purposes. Section four summarizes research results. Finally, conclusions are made in section four.

2. Proposed Approach

In robust design, the Taguchi method adopts a fractional factorial experimental design called an orthogonal array (OA), which reduces the number of experiments under permissive reliability. In an OA, the columns are pairwise orthogonal. That is, for every pair of columns, all combinations of factor levels occur an equal number of times. The columns of the OA represent factors to be investigated, while the rows denote individual experiments. The quality response can be divided into three main types involving: (1) the smaller-the-better (STB) type response, in which the response is continuous, nonnegative, and its most desired value is zero; (2) the nominal-the-best (NTB) type response, in which the response is continuous, nonnegative, and its target is nonzero and finite; and (3) the larger-the-better (LTB) type response, which can be transformed into the STB type by considering the reciprocal of the response. The Taguchi method employs the S/N ratio as a quality measure. Regardless of the response type, the optimal factor level is the factor level which maximizes the S/N ratio, η. That is, the objective function to be maximized is

for the STB type response:

$$\eta = -10\log_{10}(\frac{1}{n}\sum_{i=1}^{n} y_i^2) \quad (1)$$

for the NTB type response

$$\eta = 10\log_{10}\frac{\mu^2}{\sigma^2} \quad (2)$$

for the LTB type response

$$\eta = -10\log_{10}(\frac{1}{n}\sum_{i=1}^{n}(\frac{1}{y_i^2})) \quad (3)$$

where n is the number of replicates for y, μ is the response mean and σ is the standard deviation. Based on the above, the proposed approach for solving the multi-response problem in the Taguchi method is outlined in the following steps:

***Step* 1**:

Let r be the number of responses in an OA. Let η_j (j =1, ... , r) be the S/N ratio of response j. Then, calculate η_j for all j values using a proper formula from Eqs. (1-3).

***Step* 2**:

Assume a process factor l is assigned at K levels. Let η_{jlk} be the sum of η_js for the experiments at level k (k = 1, ..., K) of factor l, and let $\bar{\eta}_{jlk}$ be the average of η_{jlk}. Calculate $\bar{\eta}_{jlk}$ of each factor level for all responses.

***Step* 3**:

Let w_{jlk} be the weight of level k for factor l from response j, which is estimated as

$$w_{jlk} = \begin{cases} \dfrac{\max_k \bar{\eta}_{jlk}}{\bar{\eta}_{jlk}} & \text{for the STB type response} \\ \dfrac{\bar{\eta}_{jlk}}{\max_k \bar{\eta}_{jlk}} & \text{for the NTB and LTB type responses} \end{cases} \quad (4)$$

Calculate w_{jlk} values of factor l from each response j. The value of w_{jlk} surely lies between zero and one. Let $\bar{w}_{lk}$ be the average of w_{jlk} over all responses. Estimate the $\bar{w}_{lk}$ values for all levels of factor l. Typically, larger $\bar{w}_{lk}$ indicates better performance. Consequently, identify the factor level corresponding to the maximum of $\bar{w}_{lk}$ (k =1, 2, … , K) as the optimal level of factor l.

Step 4:

Calculate the anticipated improvement in each response due to setting process factors at optimal levels and compare the total anticipated improvements with those obtained in previous studies. The anticipated improvement is calculated as the S/N ratios at optimal factor level minus the S/N ratio at initial factor level

3. Illustrations

The following three previously studied case-studies are employed to illustrate the proposed approach.

3.1. Optimization of a Gear Hobbing Operation

Jeyapaul *et al*. [14] used genetic algorithm to investigate the effect of machining parameters on multiple performance characteristics of a gear hobbing operation. The objective was to determine the levels of machining parameters which optimize the profile and helix errors. Four STB type quality responses were selected, namely: left profile (LP) error, right profile (RP) error, left helix (LH) error, and right helix (RH) error. Six process-controllable factors were investigated, including: direction of hobbing (A, two levels), number of passes (B, two levels), source of hob (C, two levels), feed (D, three levels), speed (E, three levels), and job run out (F, three levels). The initial factor level was $A_1B_2C_2D_2E_1F_3$. The L_{18} array, which contains 18 experiments, was then selected for conducting the experiments. However, the L_{18}

array can accommodate one factor at two levels and seven factors, each at three levels. Thus, the two factors B and C were combined into one factor (BC) of B_1C_1, B_2C_1, and B_2C_2 for level 1, 2, and 3, respectively. The proposed approach is implemented as follows:

Step 1:

Four STB type responses (LP error, RP error, LH error, and RH error) are considered in this case study (r = 4). Let response j (j = 1, ..., 4) denote LP error, RP error, LH error, and RH error, respectively. The S/N ratio, η_j, of response j for each experiment is calculated using Eq. (1) for all j values and shown in Table 1 for all of the 18 experiments in L_{18} array.

Table 1. The S/N ratios for gear hobbing operation.

Exp.	Control factor*								S/N ratio (dB)			
	A	BC	D	E	F	*e*	*e*	*e*	LP error	RP error	LH error	RH error
1	1	1	1	1	1	1	1	1	-37.2117	-37.3816	-33.7785	-33.1746
2	1	1	2	2	2	2	2	2	-37.5854	-37.4182	-30.5984	-32.5739
3	1	1	3	3	3	3	3	3	-37.4127	-37.2801	-34.5825	-34.3794
4	1	2	1	1	2	2	3	3	-37.4870	-37.7535	-35.8931	-35.1224
5	1	2	2	2	3	3	1	1	-37.5440	-37.6162	-38.3929	-35.6160
6	1	2	3	3	1	1	2	2	-37.1266	-37.3865	-31.2694	-32.5408
7	1	3	1	2	1	3	2	3	-37.5136	-37.1455	-34.7481	-35.6156
8	1	3	2	3	2	1	3	1	-37.7367	-37.4789	-35.0478	-33.1795
9	1	3	3	1	3	2	1	2	-37.8079	-37.1797	-35.3629	-35.9544
10	2	1	1	3	3	2	2	1	-37.3458	-37.7101	-32.5646	-33.5220
11	2	1	2	1	1	3	3	2	-37.4196	-37.9687	-34.3192	-31.3069
12	2	1	3	2	2	1	1	3	-37.1512	-37.5440	-32.4770	-29.7669
13	2	2	1	2	3	1	3	2	-37.5158	-37.4474	-30.9765	-31.1584
14	2	2	2	3	1	2	1	3	-37.6769	-37.4492	-32.3626	-31.6810
15	2	2	3	1	2	3	2	1	-37.2493	-37.4868	-32.7771	-32.1360
16	2	3	1	3	2	3	1	2	-37.5777	-37.9395	-33.3351	-31.0950
17	2	3	2	1	3	1	2	3	-37.5474	-37.7383	-32.8354	-31.9022
18	2	3	3	2	1	2	3	1	-37.6057	-37.1483	-34.2558	-33.5637

Step 2:

First, the process factor A is chosen; it is assigned at two levels; A1 and A2. The $\bar{\eta}_{jA_1}$ and $\bar{\eta}_{jA_2}$; the average S/N ratios for A1 and A2, respectively, are estimated from each response j and shown in Table 2. Similarly, the S/N ratio averages for the levels of factors B to F are calculated from each response j and also displayed in Table 2 for all j values.

Table 2. The S/N ratio averages for gear hobbing operation.

Response (dB)	Factor (l) / Level (k)*	A	B	C	D	E	F
LP error	1	-37.4917	**-37.3544**	**-37.3938**	-37.4419	**-37.4538**	**-37.4257**
	2	**-37.4544**	-37.5324	-37.6315	-37.5850	-37.4859	-37.4646
	3				**-37.3922**	-37.4794	-37.5289
RP error	1	**-37.4045**	-37.5504	-37.5368	-37.5629	-37.5848	**-37.4133**
	2	-37.6036	**-37.4808**	**-37.4384**	-37.6116	**-37.3866**	-37.6035
	3				**-37.3376**	-37.5407	-37.4953
LH error	1	-34.4082	**-33.0534**	**-33.3327**	-33.5493	-34.1610	-33.4556
	2	**-32.8781**	-33.9381	-34.2642	-33.9261	-33.5748	**-33.3548**
	3				**-33.4541**	**-33.1937**	-34.1192
RH error	1	-34.2396	**-32.454**	**-32.7482**	-33.2813	-33.2661	-32.9804
	2	**-31.7925**	-33.2971	-33.5517	**-32.7099**	-33.0491	**-32.3123**
	3				-33.0569	**-32.733**	-33.7554

* Optimal factor levels for each response are identified by boldtype.

As mentioned earlier, B_1C_1, B_2C_1, and B_2C_2 are assigned at levels 1, 2, and 3, respectively, for the combined factor BC. The calculated S/N ratio averages for levels 1, 2, and 3 of factor BC from LP error are -37.3544,

-37.4333, and -37.6315 dB, respectively. The average S/N ratio for B_1 is equal to the average S/N ratio for level 1 of factor BC (= -37.3544 dB), while the average S/N ratio for B_2 (= -37.5324 dB) is estimated as the average of the S/N ratios averages for levels 2 and 3 of factor BC. Conversely, the average S/N ratio for C_1 (= -37.3938 dB) is calculated as the average of the S/N ratio averages for levels 2 and 3 of factor BC, while the average S/N ratio for C_2 (= -37.6315 dB) is equal to the average S/N ratio for level 3. In a similar manner, the S/N ratio averages for the two levels of factors B and C are estimated from each of the other three responses. Finally, the S/N ratio averages for the levels of factors D to E are calculated from each response *j* as the sum of the S/N ratios at each factor level divided by six, which is the number of experiments at that level. In Table 2, using the Taguchi method the combination of optimal factor levels for each of the LP error, RP error, LH error, and RH error is $A_2B_1C_1D_3E_1F_1$, $A_1B_2C_2D_3E_2F_1$, $A_2B_1C_1D_3E_3F_2$ and $A_2B_1C_1D_2E_3F_2$, respectively. Obviously, there exists a conflict among these combinations regarding the optimal factor levels for the four responses simultaneously.

Step 3:

The level weights for factors A to F are calculated from each response *j* using Eq. (4) and displayed in Table 3 for all *j* values.

Table 3. The level weights for gear hobbing operation.

Response	Factor (*l*) / Level (*k*)	A	B	C	D	E	F
LP error	1	0.9990	1.0000	1.0000	0.9987	1.0000	1.0000
	2	1.0000	0.9953	0.9937	0.9949	0.9991	0.9990
	3				1.0000	0.9993	0.9972
RP error	1	1.0000	0.9981	0.9974	0.9940	0.9947	1.0000
	2	0.9947	1.0000	1.0000	0.9927	1.0000	0.9949
	3				1.0000	0.9959	0.9978
LH error	1	0.9555	1.0000	1.0000	0.9972	0.9717	0.9970
	2	1.0000	0.9739	0.9728	0.9861	0.9886	1.0000
	3				1.0000	1.0000	0.9776
RH error	1	0.9285	1.0000	1.0000	0.9828	0.9840	0.9797
	2	1.0000	0.9747	0.9761	1.0000	0.9904	1.0000
	3				0.9895	1.0000	0.9572
Level weight*	1	0.9708	**0.9995**	**0.9993**	0.9932	0.9876	0.9942
	2	**0.9987**	0.9860	0.9856	0.9934	0.9946	**0.9985**
	3				**0.9974**	**0.9988**	0.9825

* Optimal factor levels are identified by boldtype.

The level weights for factor A, w_{jA1} and w_{jA2}, from LP error (*j* = 1) are calculated as follows. In Table 2, the $\overline{\eta}_{1A_1}$ and $\overline{\eta}_{1A_2}$ from LP error are calculated to be -37.4917 and -37.4544 dB, respectively. The largest S/N ratio, $\overline{\eta}_{A1}$, for factor A from LP error equals -37.4544 dB. The w_{1A1} is estimated as 0.9990 (= -37.4544/-37.4917), while w_{1A2} is calculated as 1.00 (= -37.4544/-37.4544). The w_{jA1} and w_{jA2} values from each of RP error, LH error, and RH error are estimated similarly. The average level weights for factor A are calculated from all of the four responses. It is obtained that $\overline{w}_{A_1}$ equals 0.9708, while $\overline{w}_{A_2}$ is 0.9987. As a result, the optimal level of factor A is determined to be A_2. Similar calculations are made for factors B to F and their optimal levels are fixed at B_1, C_1, D_3, E_3, and F_2, respectively. However, the combination of optimal factor levels was calculated as $A_1B_2C_2D_1E_1F_3$ by using genetic algorithm [14].

Step 4:

The anticipated improvement in each response, which is calculated from Table 2 as the average S/N ratios at optimal factor levels ($A_2B_1C_1D_3E_3F_2$) minus the average S/N ratios at initial factor levels ($A_1B_2C_2D_1E_1F_3$), is displayed in Table 4 for the four responses. The anticipated improvements from use of the genetic algorithm are also listed in Table 4.

It can be seen in Table 4 that the proposed approach results in a total anticipated improvement of 11.8005 dB, which is about three times that gained using the genetic algorithm (= 4.1499 dB).

3.2. Optimization of a polysilicon deposition process

This case study was considered by Phadke [4] and it aimed at improving the quality of a polysilicon process. Three quality responses were selected including: surface defects (the STB type response), thickness (the NTB type response, target is 3600 Å), and deposition rate (the LTB type response). The surface defects were considered as the key quality problem that causes significant scrap.

Table 4. Anticipated improvements for gear hobbing operation.

Response (dB)	Starting condition (I)	Optimal condition (II)		Anticipated improvement (II) - (I)	
		Genetic algorithm [14]	Proposed approach	Genetic algorithm [14]	Proposed approach
LP error	-37.8581	-37.4917	-37.1735	0.3664	0.6846
RP error	-37.4952	-37.4045	-37.6525	0.0907	-0.1573
LH error	-36.6009	-34.4082	-31.0508	2.1927	5.5501
RH error	-35.7397	-34.2396	-30.0166	1.5001	5.7231
Total anticipated improvement (dB)				4.1499	11.8005

Six process controllable factors were selected at three levels each; the factors were: deposition temperature (A), deposition pressure (B), nitrogen flow (C), silane flow (D), settling time (E), and cleaning method (F). The combination of initial factor levels was selected as $A_2B_2C_1D_3E_1F_1$. The orthogonal array L_{18} was selected for the experiment design. The proposed approach for the polysilicon process was implemented as follows:

Step 1:

Three responses are considered in this case study. The S/N ratios for surface defects (STB type), thickness (NTB type), and deposition rate (LTB type) are estimated for each experiment using Eqs. (1), (2), and (3), respectively, and listed in Table 5 for all of the 18 experiments.

Table 5. The S/N ratios of each response for polysilicon process.

Exp.	Control factor*								S/N ratio (dB)		
	e	A	B	C	D	E	*e*	F	Surface defects	Thickness	Deposition rate
1	1	1	1	1	1	1	1	1	0.51	35.22	23.23
2	1	1	2	2	2	2	2	2	-37.30	35.76	31.27
3	1	1	3	3	3	3	3	3	-45.17	36.02	32.34
4	1	2	1	1	2	2	3	3	-25.76	42.25	31.15
5	1	2	2	2	3	3	1	1	-62.54	21.43	37.27
6	1	2	3	3	1	1	2	2	-62.23	32.91	33.89
7	1	3	1	2	1	3	2	3	-59.88	21.39	37.68
8	1	3	2	3	2	1	3	1	-71.69	22.84	40.46
9	1	3	3	1	3	2	1	2	-68.15	30.60	41.21
10	2	1	1	3	3	2	2	1	-3.47	26.85	27.89
11	2	1	2	1	1	3	3	2	-5.08	38.80	26.02
12	2	1	3	2	2	1	1	3	-54.85	38.06	31.82
13	2	2	1	2	3	1	3	2	-49.38	32.07	34.50
14	2	2	2	3	1	2	1	3	-36.54	43.34	33.20
15	2	2	3	1	2	3	2	1	-64.18	37.44	34.76
16	2	3	1	3	2	3	1	2	-27.31	31.86	37.71
17	2	3	2	1	3	1	2	3	-71.51	22.01	40.45
18	2	3	3	2	1	2	3	1	-72.00	18.42	39.22

* Empty column for error.

Step 2:

Let response *j* where *j* equals 1, 2, and 3 denote surface defects, thickness, and deposition rate, respectively. The average S/N ratios, $\bar{\eta}_{jl_1}$, $\bar{\eta}_{jl_2}$, and $\bar{\eta}_{jl_3}$, for the three levels of each factor are estimated from each response *j* and displayed in Table 6. For illustration, the $\bar{\eta}_{1A_1}$ for A_1 from surface defects response, -24.23 dB, is calculated as the summation of the S/N ratios of experiments 1, 2, 3, 10, 11, and 12, divided by six. In a similar manner, the $\bar{\eta}_{1A_2}$ and $\bar{\eta}_{1A_3}$ values of -50.11 and -61.76, respectively, are calculated from surface defects. Similarly, the $\bar{\eta}_{jA_1}$, $\bar{\eta}_{jA_2}$, and $\bar{\eta}_{jA_3}$ are calculated from response *j* equal to 2 and 3 for thickness and deposition rate, respectively. Finally, the average S/N ratios for the three levels of factors B to F are estimated from each response *j* for all *j* values.

In Table 6, the optimal factor levels for surface defects, thickness, and deposition rate using the Taguchi method are decided as $A_1B_1C_1D_1E_2F_2$, $A_1B_3C_1D_2E_2F_3$, and $A_3B_3C_2D_3E_3F_3$, respectively. Obviously, a conflict exists among the optimal factor levels for the three responses. Thus, Phadke [4] selected the combination of optimal factor levels by engineering judgment as $A_1B_2C_1D_3E_2F_2$.

Step 3:

The w_{jlk} and $\bar{w}_{lk}$ values of each factor level are calculated from each response *j* using Eq. (4) for all *j* values; they are listed in Table 7 for all factor levels. For example, the w_{1A1}, w_{1A2}, and w_{1A3} of the three levels of factor A are calculated from surface defects as 1.00 (= -

Table 6. The S/N ratio averages for polysilicon process.

Response (dB)	Factor (l) / Level (k)*	A	B	C	D	E	F
Surface defects	1	**-24.23**	**-27.55**	**-39.03**	**-39.20**	-51.53	-45.56
	2	-50.11	-47.44	-55.99	-46.85	**-40.54**	**-41.58**
	3	-61.76	-61.10	-41.07	-50.04	-44.03	-48.95
Thickness	1	**35.12**	31.61	**34.39**	31.68	30.52	27.04
	2	34.91	30.70	27.86	**34.70**	**32.87**	33.67
	3	24.52	**32.24**	32.30	28.16	31.16	**33.85**
Deposition rate	1	28.76	32.03	32.80	32.21	34.06	33.81
	2	34.13	34.78	**35.29**	34.53	33.99	34.10
	3	**39.46**	**35.54**	34.25	**35.61**	**34.30**	**34.44**

* Optimal factor levels for each response are identified by boldtype.

24.23/-24.23), 0.4835 (= -24.23/-50.11) and 0.3923 (= -24.23/-61.76), respectively. Similarly, the w_{jA1}, w_{jA2} and w_{jA3} are estimated from each response j for j equals 2 and 3 for thickness and deposition rate, respectively. The $\overline{w}_{A_1}$, $\overline{w}_{A_2}$ and $\overline{w}_{A_3}$ are then calculated from the three responses as 0.9096, 0.7808, and 0.6968, respectively. Since the maximum level weight for factor A corresponds to A_1, the optimal level of factor A is decided at A_1. Similarly, the w_{jlk} and $\overline{w}_{lk}$ values are estimated for factors B to F, where their optimal levels are estimated as B_1, C_1, D_1, E_2, and F_2, respectively.

Table 7. The level weights for polysilicon process.

Response	Factor (l) / Level (k)	A	B	C	D	E	F
Surface defects	1	1.0000	1.0000	1.0000	1.0000	0.7867	0.9126
	2	0.4835	0.5807	0.6971	0.8367	1.0000	1.0000
	3	0.3923	0.4509	0.9503	0.7834	0.9207	0.8494
Thickness	1	1.0000	0.9805	1.0000	0.9130	0.9285	0.7988
	2	0.9940	0.9522	0.8101	1.0000	1.0000	0.9947
	3	0.6982	1.0000	0.9392	0.8115	0.9480	1.0000
Deposition rate	1	0.7288	0.9012	0.9294	0.9045	0.9930	0.9817
	2	0.8649	0.9786	1.0000	0.9697	0.9910	0.9901
	3	1.0000	1.0000	0.9705	1.0000	1.0000	1.0000
Level weight*	1	**0.9096**	**0.9606**	**0.9859**	**0.9392**	0.9027	0.8977
	2	0.7808	0.8372	0.8357	0.9355	**0.9970**	**0.9949**
	3	0.6968	0.8170	0.9534	0.8650	0.9562	0.9498

* Optimal factor levels are identified by boldtype.

Step 4:

The anticipated improvements in surface defects, thickness, and deposition rate are calculated at $A_1B_1C_1D_1E_2F_2$ and listed in Table 8. The sum of the weighted normalized quality loss [6], PCA [9], and DEAR [11] approaches discussed this case study and their anticipated improvements are also summarized and compared in Table 8.

Table 8. Anticipated improvement for polysilicon process.

Response (dB)	Starting condition (I)	Optimal condition (II): Engineering judgment [4]	Weighted quality loss [6]	PCA [9]	DEAR [11]	Proposed approach	Anticipated improvement (II) - (I): Engineering judgment [4]	Weighted quality loss [6]	PCA [9]	DEAR [11]	Proposed approach
Surface defects	-56.69	-19.84	-24.22	-2.29	1.20	14.68	36.85	32.47	54.40	57.89	71.37
Thickness	29.95	36.79	40.24	41.23	41.32	41.77	6.84	10.29	11.28	11.37	11.82
Deposition rate	34.97	29.60	32.44	27.21	27.21	23.32	-5.37	-2.53	-7.76	-7.76	-11.66
Total anticipated improvement (dB)							38.32	40.23	57.92	61.5	71.53

In Table 8, using the proposed approach, the surface defects are reduced by 71.37 dB and thickness is improved by 11.82 dB; these are the largest among the anticipated improvements for all examined approaches. Deposition rate, however, is decreased by 11.66 dB, which is the worst anticipated improvement because it is correlated to surface defects. Despite that, the proposed approach contributes the largest total anticipated improvement of 71.53 dB.

3.3. Optimization of a Plasma-Enhanced Chemical Vapour Deposition

This case study was conducted by Tong *et al*. [6] to improve the performance of a plasma-enhanced chemical vapour deposition process. Two quality responses were studied, including deposition thickness (DT, the target is 1000 Å) and refractive index (RI, the target value is 2). Eight controllable process factors (A to H) were investigated. Factor A was decided at two levels, while factors B to H were each selected at three levels. The L_{18} array was selected for experimental design. The experiments were performed by the Industrial Technology Research Institute, Taiwan. The proposed approach can be briefly described as follows:

Step 1:

The S/N ratios of each DT and RI, which are NTB type responses, are calculated using Eq. (2) for each experiment; values for all of the 18 experiments are shown in Table 9.

Table 9. Experimental data for plasma-enhanced process.

Exp.	Control factors								Deposition thickness (DT)			Refractive index (RI)		
	A	B	C	D	E	F	G	H	Average	Standard deviation	S/N ratio	Average	Standard deviation	S/N ratio
1	1	1	1	1	1	1	1	1	730.60	62.4884	21.36	2.03	0.0802	28.07
2	1	1	2	2	2	2	2	2	874.20	25.8979	30.57	2.22	0.0412	34.64
3	1	1	3	3	3	3	3	3	967.20	52.1076	25.37	2.61	0.1026	28.11
4	1	2	1	1	2	2	3	3	800.80	34.6222	27.28	2.02	0.0557	31.19
5	1	2	2	2	3	3	1	1	789.20	113.1159	16.87	1.97	0.0751	28.36
6	1	2	3	3	1	1	2	2	796.20	48.6487	24.28	1.88	0.0675	28.89
7	1	3	1	2	1	3	2	3	909.80	194.8017	13.39	1.89	0.0873	26.73
8	1	3	2	3	2	1	3	1	648.80	93.5452	16.82	1.78	0.0351	34.08
9	1	3	3	1	3	2	1	2	646.60	93.5698	16.79	1.70	0.0197	38.69
10	2	1	1	3	3	2	2	1	1013.40	112.0750	19.13	1.97	0.0838	27.44
11	2	1	2	1	1	3	3	2	1493.60	327.3894	13.18	1.83	0.1655	20.86
12	2	1	3	2	2	1	1	3	900.60	59.0788	23.66	1.90	0.0559	30.60
13	2	2	1	2	3	1	3	2	902.40	94.5637	19.59	1.83	0.0551	30.42
14	2	2	2	3	1	2	1	3	824.80	85.2508	19.71	2.04	0.0610	30.50
15	2	2	3	1	2	3	2	1	792.60	104.4811	17.60	2.10	0.0888	27.46
16	2	3	1	3	2	3	1	2	814.60	146.6332	14.89	2.19	0.0632	30.80
17	2	3	2	1	3	1	2	3	818.00	43.9431	25.40	1.91	0.0165	41.29
18	2	3	3	2	1	2	3	1	738.80	36.2036	26.20	2.02	0.0635	30.06

Step 2:

Let *j* of 1 and 2 represent DT and RI, respectively. The $\bar{\eta}_{jA_1}$ and $\bar{\eta}_{jA_2}$ for factor A, and the $\bar{\eta}_{jl_1}$, $\bar{\eta}_{jl_2}$, and $\bar{\eta}_{jl_3}$ for factors B to H are estimated from each response *j* and displayed in Table 10 for all *j* values. In this table, the optimal combinations of factor levels using the Taguchi method are $A_1B_1C_3D_2E_2F_2G_2H_3$ and $A_1B_3C_2D_1E_3F_1G_1H_3$ for DT and RI, respectively. Clearly, there is a conflict among the optimal factor levels for the two responses.

Step 3:

The values of w_{jlk} are estimated for all factor levels from each response *j* using Eq. (4). All w_{jlk} values are shown in Table 11. The $\bar{w}_{A_1}$ and $\bar{w}_{A_2}$ for factor A and $\bar{w}_{l1}$, $\bar{w}_{l_2}$, and $\bar{w}_{l_3}$ for factors B to H are then estimated from the two responses. It is found that the combination of optimal factor levels is $A_1B_3C_3D_2E_2F_2G_2H_3$. However, the combination of optimal factor levels using the sum of the weighted normalized quality losses [6] was $A_1B_3C_2D_2E_2F_2G_2H_3$.

Step 4:

The anticipated improvements in DT and RI are calculated at $A_1B_3C_3D_2E_2F_2G_2H_3$ and shown with the anticipated improvements by the sum of the weighted normalized quality losses approach are also displayed and compared in Table 11.

In this table, it is obvious that the proposed approach results in better anticipated improvements than the sum of the weighted normalized quality losses approach in both the DT and RI responses.

Table 10. The S/N ratio averages for plasma-enhanced process.

Response (*j*)	Factor (*l*) / Level (*k*) *	A	B	C	D	E	F	G	H
Deposition Thickness (DT)	1	**21.41**	**22.21**	19.27	20.27	19.69	21.85	18.88	19.66
	2	19.93	20.89	20.43	**21.71**	**21.80**	**23.28**	**21.73**	19.88
	3		18.91	**22.32**	20.03	20.53	16.89	21.41	**22.47**
Refractive index (RI)	1	**30.97**	28.29	29.11	**31.26**	27.52	**32.23**	**31.17**	29.24
	2	29.94	29.47	**31.62**	30.13	31.46	32.09	31.07	30.72
	3		**33.61**	30.64	29.97	**32.39**	27.05	29.12	**31.40**

* Optimal factor levels for each response are identified by boldtype.

Table 11. Anticipated improvement for plasma-enhanced process.

Response (dB)	Starting condition (I)	Optimal condition (II)		Anticipated improvement (II) - (I)	
		Weighted quality loss [6]	Proposed approach	Weighted quality loss [6]	Proposed approach
Deposition thickness (DT)	21.62	25.44	28.93	3.77	7.31
Refractive index (RI)	32.09	37.93	38.19	5.84	6.10
Total anticipated improvement (dB)				9.61	13.41

4. Research Results

The proposed approach has been adopted for solving the multi-response problem in the Taguchi method for three case studies. It is found that the proposed approach has the largest anticipated improvement in reducing profile and helix variation for gear hobbing operation, in dramatically improving the performance of polysilicon process, and effectively optimizing the plasma-enhanced chemical vapour deposition. Moreover, several advantages of the proposed approach have been noticed, including: (1) it is not based on rigid assumptions as PCA does, (2) it is simple since it does not require advanced statistical skills or *a priori* information about the response weights or importance, and (3) it requires minimal computational effort. Definitely, these advantages shall attract product/process engineering to adopt it for solving the multi-response problem in the Taguchi method in a wide range of engineering applications.

5. Conclusions

This research proposed a simple, yet very effective, approach for solving the multi-response problem in the Taguchi method. In this approach, the S/N ratio of each response is calculated. Then, the average S/N ratio for each factor level from all responses is estimated and used to decide the optimal levels for all process factors. Three case studies were provided for illustration, and the proposed approach provided the largest total anticipated improvement in multi-responses among all used approaches, including PCA, DEAR, and genetic algorithm. The primary advantage of this approach is that it is characterized by simple and straightforward calculations without the need for statistical skills or *a priori* information about response weights or importance. In conclusion, the simplicity and effectiveness of this approach provides great assistance to practitioners for solving the multi-response problems in a wide range of applications on the Taguchi method.

References

[1] G. Taguchi, "*Taguchi Methods. Research and Development. Vol.* 1," Dearborn, MI, American Suppliers Institute Press, 1991.

[2] K. Bilen, S. Yapici, C.A. Celik, "Taguchi approach for investigation of heat transfer from a surface equipped with rectangular blocks," *Energy Conversion and Management,* Vol. 42 , 2001, 951-961.

[3] A.R. Khoei, I. Masters, D.T. Gethin, "Design optimization of Aluminium recycling processes using Taguchi technique," *Journal of Materials Processing Technology*, Vol.127, 2002, 96-106.

[4] M.S. Phadke, "Quality Engineering Using Robust Design," NJ, Prentice-Hall, Englewood Cliffs, 1989.

[5] G.H. Shiau, "A study of the sintering properties of iron ores using the Taguchi's parameter design," *Journal of the Chinese Statistical Association,* Vol. 28 , 1990, 253-275.

[6] L.I. Tong, C.T. Su, C.H. Wang The optimization of multi-response problems in the Taguchi method. *International Journal of Quality and Reliability Management,* Vol. 14, No.4 , 1997, 367-380.

[7] N. Logothetis, A. Haigh, "Characterizing and solving multi-response processes by the Taguchi method," *Quality and Reliability Engineering International*, Vol. 4, No. 2 , 1988, 159-169.

[8] J.J. Pignatello,"Strategies for robust multiresponse quality engineering," *IIE Transactions*, Vol. 25, 1993, 5-15.

[9] C.T. Su, L.I. Tong, “Multi-response robust design by principal component analysis,” *Total Quality Management*, Vol. 8, No. 6 , 1997, 409-416.

[10] J. Antony, “Multi-response optimization in industrial experiments using Taguchi’s quality loss function and principal component analysis,” *Quality and Reliability Engineering International*, Vol.16 , 2000, 3-8.

[11] H.C. Liao, Y.K. Chen, “Solving multi-response problem in the Taguchi method by DEA based ranking method,” *International Journal of Quality & Reliability Management*, Vol. 19, No. 7, 2002, 825-837.

[12] Y.H.W. Wong, J.E. Beasley, “Restricting weight flexibility in DEA,” *The Journal of the Operational Research Society*, Vol. 41, 1990, 829-835.

[13] R.C. Baker, S. Talluri (1997). A closer look at the use of data envelopment analysis for technology selection. *Computers and Industrial Engineering*, Vol. 28, 1997, 101-108.

[14] R. Jeyapaul, P. Shahabudeen, K. Krishnaiah, “Simultaneous optimization of multi-response problems in the Taguchi method using genetic algorithm,” *International Journal of Advanced Manufacturing Technology*, Vol. 30, 2006, 870-878.