

# A Finite Element Calculation of Stress Intensity Factors of Cruciform and Butt Welded Joints for Some Geometrical Parameters

A. M. Al-Mukhtar <sup>a,\*</sup>, S. Henkel <sup>a</sup>, H. Biermann <sup>a</sup>, P. Hübner <sup>b</sup>

<sup>a</sup> Institute of Materials Engineering, Technische Universität Bergakademie Freiberg, Gustav-Zeuner-Straße 5, Germany

<sup>b</sup> Fachhochschule Mittweida, Mittweida, Germany

## Abstract

With welded joints, stress concentrations occur at the weld toe and at the weld root, which make these regions the points from which fatigue cracks may initiate. To calculate the fatigue life of welded structures and to analyze the progress of these cracks using fracture mechanics technique requires an accurate calculation of the stress intensity factor SIF. The existing SIFs were usually derived for one particular geometry and type of loading. In this study, the finite element method (FEM) was used to calculate the SIF. The stress intensity factors during the crack propagation phase were calculated by using the software FRANC2D, which is shown to be highly accurate, with the direction of crack propagation being predicted by using the maximum normal stress criterion. In the current work, a new analytical approach for the weld toe crack in cruciform welded joints has been used. The SIF results from FRANC2D were compared with those from the International Institute of Welding-IIW, and literature. A good correlation was obtained and the work results have bench marked which made it possible to use FRANC2D to simulate different weld geometries. The results of these comparisons are shown and the agreement is clearly well.

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Keywords: Stress Intensity Factor; FRANC2D; Welded Joints; Cruciform Welded Joint; Butt Welded Joint; IIW.

## 1. Introduction

In the application of fracture mechanics to fatigue problems, an accurate determination of the stress intensity factor (SIF) for the case under analysis is necessary to use the Paris' equation to calculate the fatigue life.

With fillet welded joints, stress concentrations occur at the weld toe and at the weld root, which make these regions the points from which fatigue cracks may initiate [1]. Therefore Shen and Clayton [2] stated that all the cracks were found to be initiated at the weld end toe, the maximum stress concentration site.

Table 1 shows some of welded joints used in this work. Discarding the major weld defects, fatigue cracks originate from the weld toe, and then propagate through the base material, or from the weld root, and then propagate through the weld throat.

Toe cracks have been considered because it is easier to observe with the naked eye as well as with dye penetration tests, they are often found in many important engineering welded structures. Moreover, there is a high stress concentration located at this point [4-5].

For fracture mechanics treatments, in spite of the fact that several SIF handbooks have been published, it is still

difficult to find solutions adequate to many welded configurations. This is mainly due to a wide variety of complex welded geometries and loading systems.

Thus, the derivation of SIF even for one type of weldment, requires detailed analysis of several parameters such as plate thickness, weld thickness, weld angle, weld toe radius and loading system.

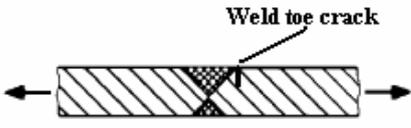
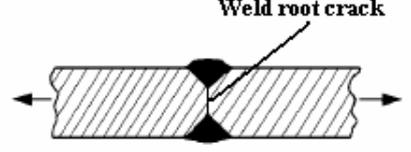
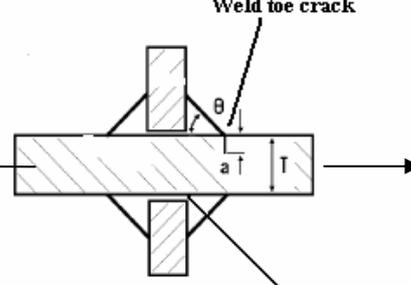
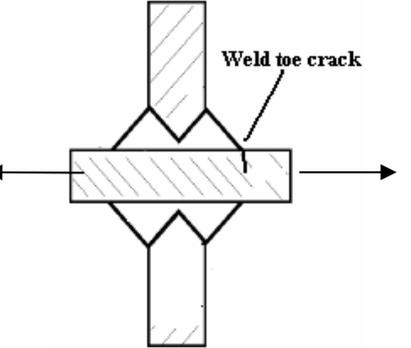
Stress intensity factors are inevitable parameter, which must be determined in fracture mechanics methods. These factors describe the fatigue action at a crack tip in terms of crack propagation.

By characterizing stable macroscopic crack growth using SIF range  $\Delta K$ , it is possible to predict crack growth rate of a crack under cyclic loading, and hence the number of cycles necessary for a crack to extend from some initial size, i.e., the size of pre-existing crack or crack-like defects, to a maximum permissible size just before catastrophic failures [6].

In this work the SIFs of cruciform joints and butt welded joints failing from weld toe have been calculated using the two-dimensional Fracture Analysis Code program FRANC2D. These results are compared with solutions from International Institute of Welding (IIW) [3], and solutions from literature.

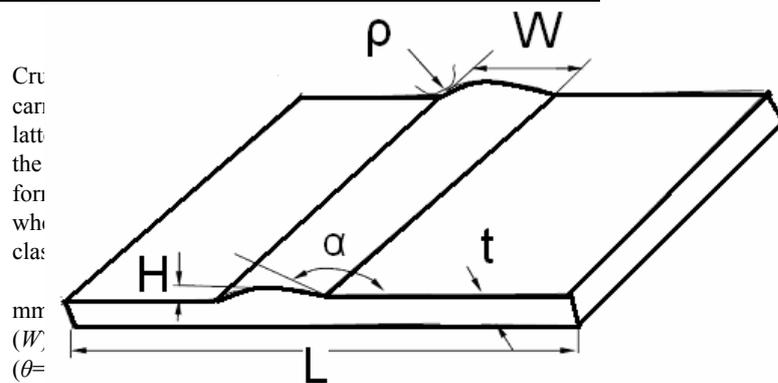
\* Corresponding author. a.almukhtar@yahoo.de.

Table 1. Fatigue cracking of some welded joints [3].

 <p style="text-align: center;"><b>Weld toe crack</b></p>	<p>Transverse loaded butt weld (X-groove or V-groove) ground flush to plate, toe crack.</p>
	<p>Transverse butt weld with weld reinforcement, toe crack.</p>
 <p style="text-align: center;"><b>Weld root crack</b></p>	<p>Transverse partial penetration butt weld. Lack of penetration (LOP) considered being as a root crack.</p>
 <p style="text-align: center;"><b>Weld toe crack</b></p> <p style="text-align: center;"><b>Weld root crack</b></p>	<p>Cruciform joint or T-joint, fillet welds or partial penetration K-butt welds, toe and root crack.</p>
 <p style="text-align: center;"><b>Weld toe crack</b></p>	<p>Cruciform joint or T-joint, K-butt welds, full penetration, toe crack.</p>

**2. Specimen Modelling**

The cruciform welded joint with equal attachment and main plate thickness ( $B/T=1$ ), is shown in Figure 1. The non-load carrying cruciform joint is made from the welding of stiffener plate (attachment plate) perpendicular to main plate (loading plate). Sheet thickness was taken as 12 mm. Because of geometrical symmetry joint, only quarter or half of the joint was considered during the calculation. The initial crack length,  $a_i$  located at weld toe, and  $h$  the weld leg length over main plate side, where  $w=2h+B$ . The weld leg length on the attached plate side is  $S$ , as shown in Figure 1. Here,  $\rho$  is the weld toe radius.



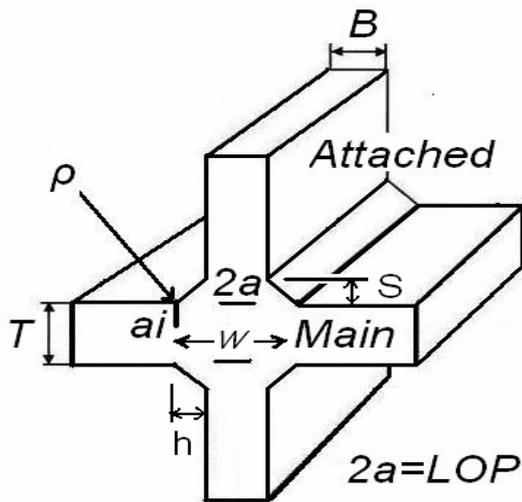


Figure 1. Cruciform fillet welded joint.

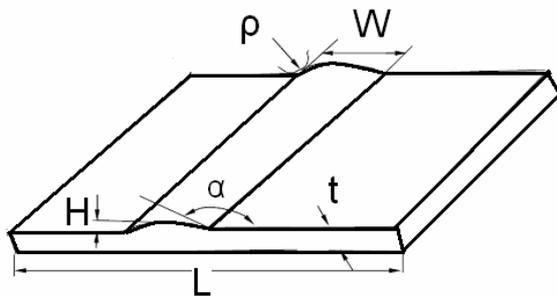


Figure 2. Weld geometry parameters at butt welded joint.

### 3. Finite Element Analysis

The finite element program FRANC2D have used in this study. The program was developed by the Cornell Fracture Group from Cornell University, USA [7].

#### 3.1. Assumption

The analysis was undertaken based on the assumption of an initially isotropic elastic material for both the base and its weld metal, in which a crack was subsequently allowed to form and grows according to a fracture criterion.

In fracture mechanics, the fatigue strength of a welded joint is not primarily governed by the strength of the base material of the joining members, hence, the governing parameters in fracture mechanics are mainly the local and global geometry of the joint only, i.e., fatigue strength is known to be closely related to the precise geometrical discontinuity of the welded joint [6]. Therefore the similar material as a quantity joint between the weld metal and a base metal has been assumed in the simulation.

#### 3.2. Mesh Description and Boundary Conditions

Figure 3 shows one of the FE meshes used in the present study, which comprises 8 noded quadrilateral elements. The boundary conditions of cruciform and butt joints are shown also in Figure 3. However, there are other possibilities to use mesh and numbers of nodes in FRANC2D program [8 - 9].

Boundary conditions were shown as the fixed sides for half or quarter models that used in this study. One end of the models was supported in the x-direction and a uniform distribution stresses were applied at the other end. To prevent the model from performing rigid body motions, one node on the side where the model is supported in the x-direction, is also locked in the y-direction, as shown in Figure 3.

#### 3.3. Symmetry

If the loading and the cracks are not considered, the cruciform and butt joints are symmetrical about both x- and y-axes. However, this is not the case when the crack is presented at weld toe. The model becomes unsymmetrical.

Quarter symmetry for cruciform joints has been used with 8 noded quadrilateral elements. Moreover, the joint can be simplified to a half model as shown in Figure 3. The difference in the SIFs solutions between the simplified half model and a quarter models for cruciform joint is shown in Figure 4. The differences are quite small.

The comparison between half and complete butt weld joints is shown in Figure 5.

The quarter and half models are therefore having a reasonable approximation of the cruciform and butt joints to estimate the SIFs of these joints.

#### 3.4. Material Properties

The material used in the present study was an extra high strength hot rolled steel with the minimum yield strength 550 MPa and the tensile strength minimum 600 MPa and maximum 760 MPa, respectively. The fatigue simulated with applied loading such that the maximum stress was maintained constant at 200 N/mm<sup>2</sup> and 104 N/mm<sup>2</sup> for cruciform and butt weld joints respectively. Values of Poisson's ratio  $\nu$  and the modulus of elasticity  $E$  were chosen as 0.293 and 210 kN/mm<sup>2</sup> respectively. Experimentally, many structures are optimized by the choice of high strength steel with the very reason for this choice is to allow for higher stresses and reduced dimensions taking benefit of the high strength material with respect to the yield criterion.

### 4. Solution Procedure

#### 4.1. Mesh Generation

A mesh generating program CASCA, which is distributed with FRANC2D, was used to create the initial mesh configuration for FRANC2D simulations. Other mesh generating programs can also be used, provided a translator is available to convert the mesh description to the FRANC2D \*.inp format [8, 9]. The procedure for creating a mesh is very straightforward, as illustrated in Figure 6. To begin with, the problem outline is first created in Figure 6a, followed by the division of sub-regions within the problem boundary. Prior to assigning the type of elements to each of the sub-regions, the boundaries for all sub-regions are divided into the required number of segments, Figure 6b. The resulting mesh for the present simulation is shown in Figure 6c.

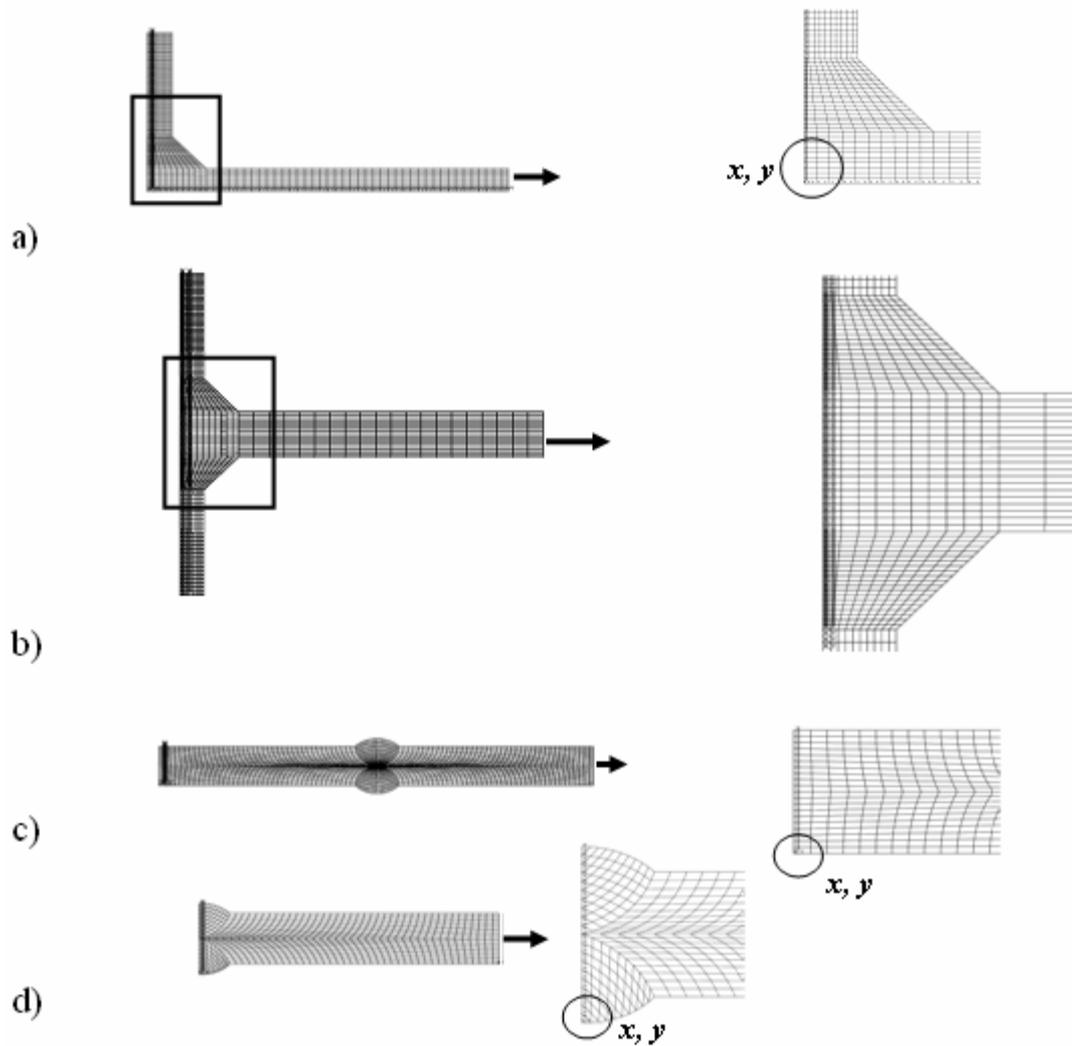


Figure 3. Mesh configuration and boundary conditions for notch cases, a) Quarter cruciform model, b) half cruciform model, c) complete butt model, d) half butt model. The edges of the models were locked in both degrees of freedom.

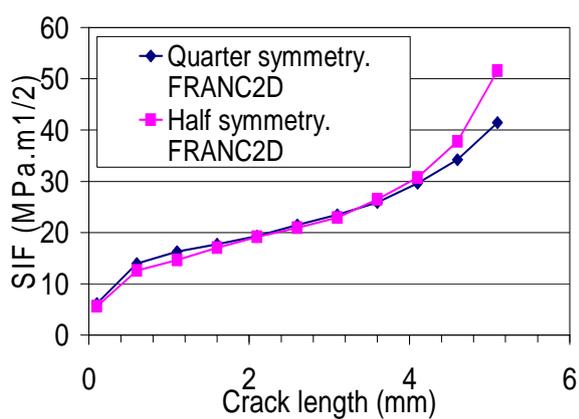


Figure 4. Comparison between half and quarter cruciform joints.

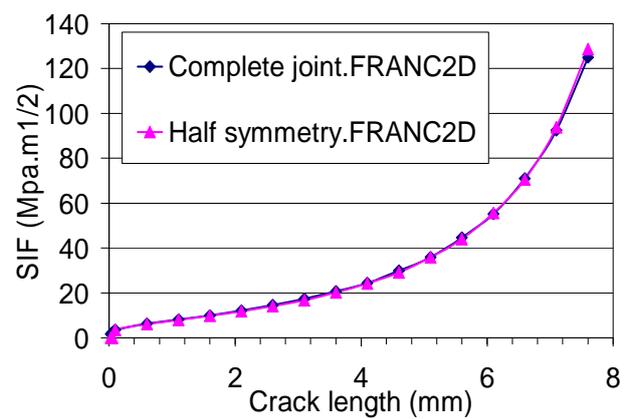


Figure 5. Comparison between half and complete butt joints.

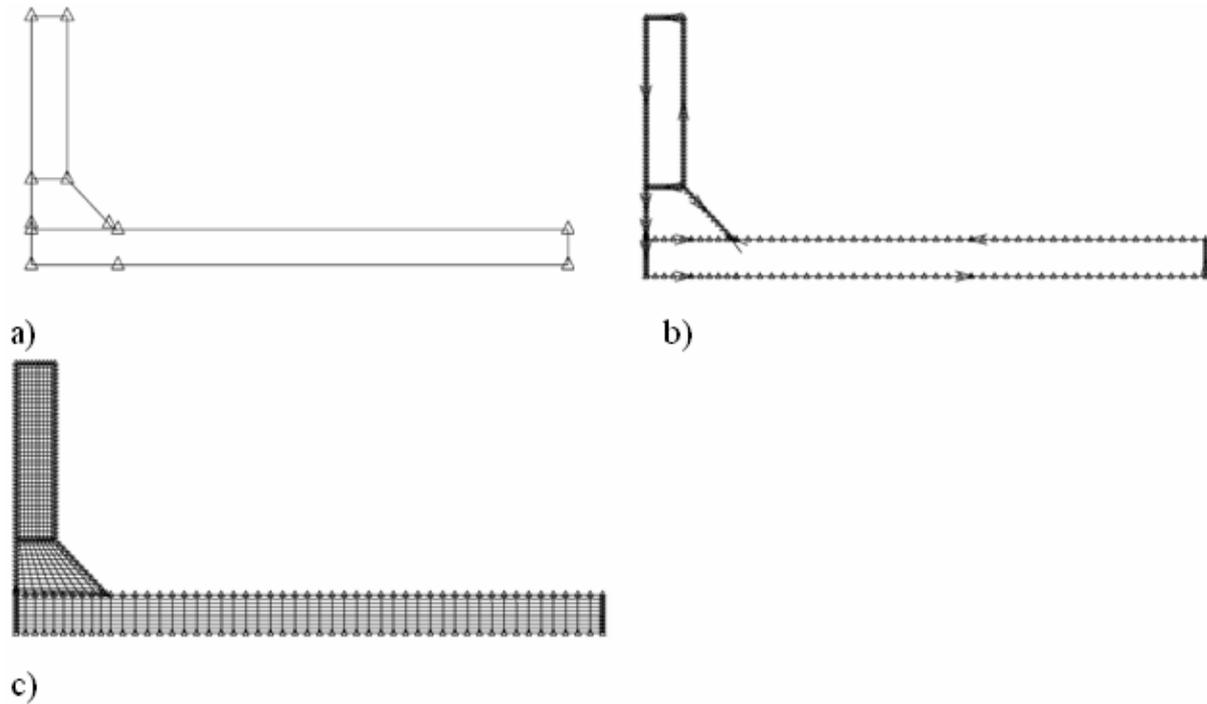


Figure 6. Mesh generating procedure, a) Geometry with subdivided areas, b) Subdivided area into segments, c) Mesh generation (CASCA) [7].

#### 4.2. Material Model

A linear elastic material model coupled to a linear elastic discrete fracture model was used in the analysis. It is possible to use an elasto-plastic material for the steel weldments, which is available in the program. But for this analysis based on fracture mechanics for which linear elastic fracture theory is considered reasonable, it was decided to restrict the analysis of SIF solutions to the consideration of a linear material behavior.

#### 4.3. Crack Propagation

A crack could propagate both from the weld toe and from the weld root (LOP). All calculations of the present work will be made on cracks propagating from the weld toe.

In order to study the capability of the program to simulate crack growth, an initial non-cohesive edge crack was placed at the weld toe of the fillet welded joints of the weld-base material interface, where it was predicted that critical tensile stresses would occur. The existence of crack-like imperfections in the welded joint is normally considered to eliminate the so-called crack initiation stage of fatigue life. Therefore, most of fatigue assessment of welded joint is concentrated on the crack growth stage of the fatigue life. Having specified the location of the crack, the program was able to predict the direction in which the crack would propagate. Prior to performing the analysis, it was necessary to specify the magnitude of crack increment and also the number of steps over which the crack would propagate. In the present study, a crack increment ( $\Delta a$ ) of 0.5 mm was chosen for all cases. The crack growth was simulated over suitable steps of increment according to plate thickness. Moreover, in this study, the crack path was not pre-selected, but crack direction was allowed to change according to the maximum tangential stress

criterion [6]. Moreover, the auto-mesh was carried out automatically.

### 5. Stress Intensity Factor Calculation

The expression for the SIF is:

$$K_I = \sigma (\pi a)^{1/2} f(b) \quad (1)$$

The applied stress is  $\sigma$  and  $f(b)$  is the correction factor. The stress intensity correction factor  $f(b)$  is a function of the crack length to thickness of propagation plate ratio. The analytical expression for  $f(b)$  can be of the tangent or secant type. It is not too obvious how to choose between the tangent and secant expression for estimating the value of parameter  $f(b)$ , although the "secant" version appears to be a bit more straightforward [10].

$$f(b) = \left( \sec \frac{\pi a}{2t} \right)^{1/2} \quad (2)$$

where the term  $\left( \frac{\pi a}{2t} \right)$  is given in radians.

Some modifications have been carried out on geometrical correction factor as shown for cruciform joint in the next section.

The FE crack growth program FRANC2D was used to calculate the opening mode SIF using fracture mechanics approach. The influence of  $K_I$  on fatigue crack growth was based on the maximum tangential stress criterion by Erdogan and Sih [11]. This criterion assumes that the predicted propagation path of the fatigue crack is perpendicular to the maximum principal stress and the crack grows under opening mode. The equivalent opening

mode (I) SIF which is used in crack growth predictions is calculated according to the same criterion.

### 5.1. Cruciform Welded Joints

For an elliptical crack at the weld toe of a fillet welded joint, the range of the SIF,  $\Delta K$  can be written as [12]:

$$\Delta K = \frac{M_k Y_u}{\phi_0} \Delta \sigma \sqrt{a} \quad (3)$$

$M_k$  is the stress concentration magnification factor,  $\Delta \sigma$  is the nominal tensile stress range applied on the main plate and  $a$ , is the crack depth [6].  $\Phi_0$  is the complete elliptical integral defined as:

$$\phi_0 = \int_0^{\pi/2} \left[ 1 - \left( 1 - \frac{a^2}{c^2} \right) \sin^2 \phi \right]^{1/2} d\phi \quad (4)$$

where  $\phi$  is parametric angle of ellipse.

In this analysis the  $M_k$ -factor functions are based on continuous edge cracks, hence, the crack aspect ratio is zero,  $a/2c = 0$ , and  $\Phi_0 = 1$ . The correction term  $Y_u$  for a double-edge crack in a plate under tensile loading, Eq. 5, given by Brown and Srawley [13] was applied:

$$Y_u = 1.98 + 0.36 \left( \frac{2a}{T} \right) - 2.12 \left( \frac{2a}{T} \right)^2 + 3.42 \left( \frac{2a}{T} \right)^3, \quad (5)$$

$$0 < \frac{2a}{T} < 0.95$$

$T$ , is the main plate thickness. For fracture mechanics treatments Maddox [12] introduced the concept of  $M_k$  as a magnification of the SIF, which would be present for a crack of the same geometry but without the presence of the weld. Further work on  $M_k$  values for cracks at weld toes has been carried out by Lie [14], Thurlbeck [15] and Bowness and Lee [16].

The formula of  $M_k$  from IIW has been used, where IIW introduced a systematic set of formulae for  $M_k$  values for different welded joints [3]:

$$M_k = C \left( \frac{a}{T} \right)^k \quad M_k \leq 1 \quad (6)$$

$$C = 0.8068 - 0.1554 \left( \frac{S}{T} \right) + 0.0429 \left( \frac{S}{T} \right)^2 + 0.0794 \left( \frac{h}{T} \right) \quad (7)$$

$$k = -0.1993 - 0.1839 \left( \frac{S}{T} \right) + 0.0495 \left( \frac{S}{T} \right)^2 + 0.0815 \left( \frac{h}{T} \right) \quad (8)$$

where  $h$  and  $S$  are the weld leg length on main and attachment plate side respectively,  $T$  is the main plate thickness and  $a$  is the initial crack length from the weld toe.

By substituting Eq. 6 and Eq. 5 in Eq. 3, SIF can be calculated for cruciform joint fail from the weld toe.

For fillet welds, the high stress concentration in weld toe is presented due to the fact that these locations rely to be sound and usually weldment contains flaws and crack-like defects. Therefore, the presence of weld toe radius inevitably will reduce these concentrations of stresses near the weld toe.

### 5.2. Butt Weld Joint

For the validation of the prediction results, the range of SIF,  $\Delta K$  for the butt welded specimens was calculated using the following empirical Eq. 9 shown below [17]:

$$\Delta K = \Delta \sigma \sqrt{\pi a} \left( 1.12 - 0.23 \left( \frac{a}{t} \right) + 10.55 \left( \frac{a}{t} \right)^2 - 21.72 \left( \frac{a}{t} \right)^3 + 30.39 \left( \frac{a}{t} \right)^4 \right) \quad (9)$$

where  $\Delta \sigma$  is the stress range,  $a$ , the crack length,  $t$  the thickness of the plate (see Figure 2). The results from Eq. 9 were compared with those obtained using FRANC2D.

## 6. Results and Discussion

### 6.1. Stress Distribution and Crack Growing

Due to stress concentration and cyclic loading, cracks may initiate and grow in the vicinity of the welds during service life even if the applied stresses are well below the yield limit. The weld toe and the weld root have high stress concentration which makes these regions the easier points from which fatigue cracks may initiate [1].

Figure 7 shows the maximum and minimum principal stress distributions for uniform load distributions of cruciform joint. Figure 8 gives the principal stresses distribution for the case of an edge crack. The stress near the weld toe was 1103 MPa which is higher than the tensile strength of materials.

The site and curved crack growth paths of continuous toe cracks were taken into account as shown in Figure 9. Normally this kind of non-load carrying welded attachments always fails at the weld toe as shown in Figure 10.

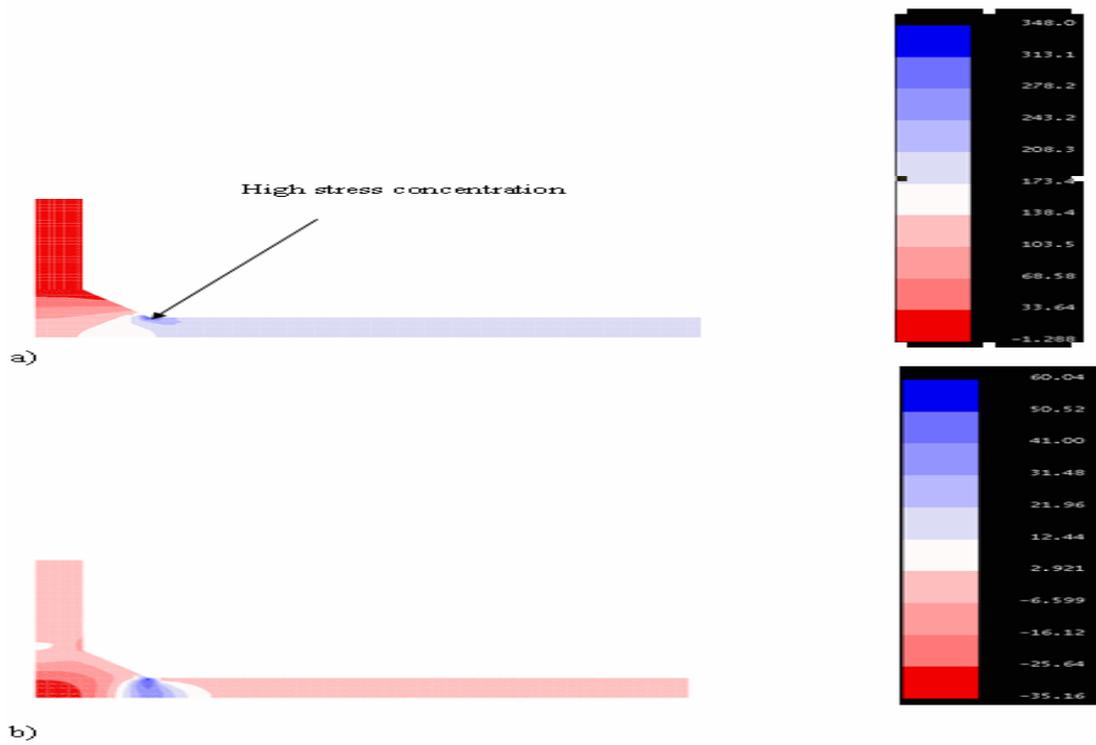


Figure 7. a) Maximum and b) Minimum principal stress distribution for cruciform joint ( $\text{N}/\text{mm}^2$ ); the stress concentration sites shown for applied load equal  $200 \text{ N}/\text{mm}^2$ .

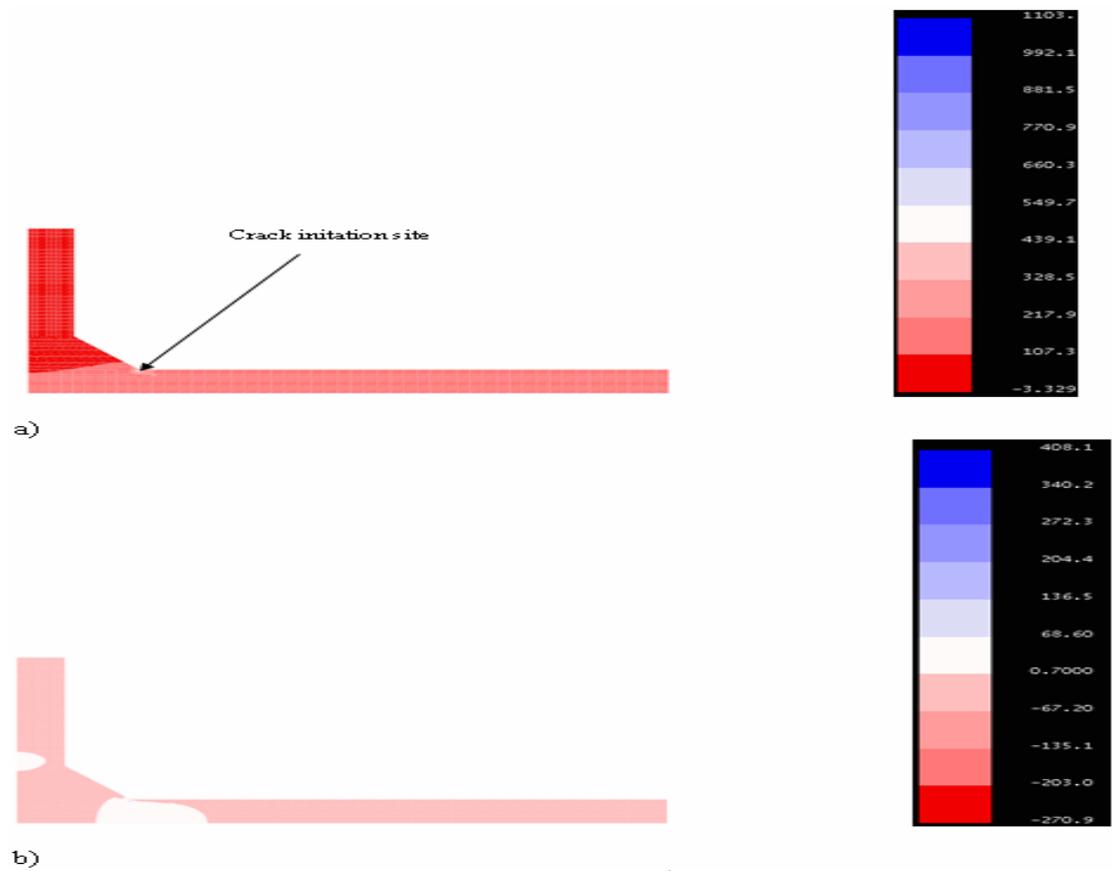


Figure 8. a) Maximum and b) Minimum principal stress distribution ( $\text{N}/\text{mm}^2$ ) in existence of edge toe crack ( $a_i=0.1 \text{ mm}$ ).

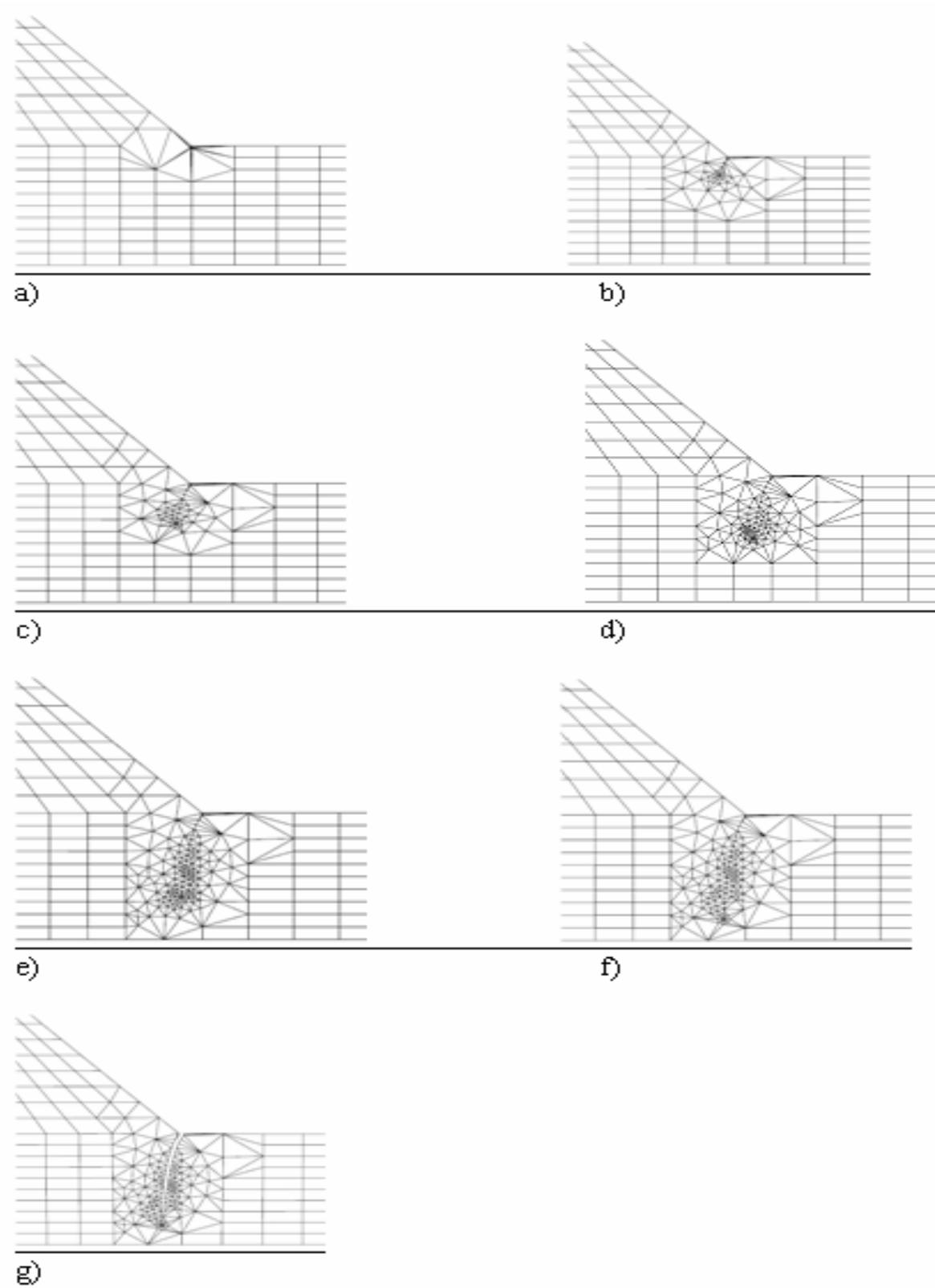


Figure 9. Sequences of crack propagation steps from the weld toe of cruciform welded joints (FRANC2D).

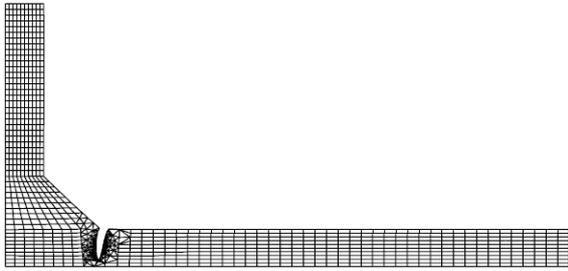


Figure 10. Crack deformation for cruciform weld joint (FRANC2D).

6.2. Butt Weld

The same steel was used for the butt weld joint. Welded toe crack has initiated and grown through the sheet thickness of 10 mm.

Figure 11 shows the modelling and boundary conditions used for butt weld to calculate SIFs. Figures 12 and 13 show the crack initiation and propagation respectively. The final crack deformation is shown in Figure 14.

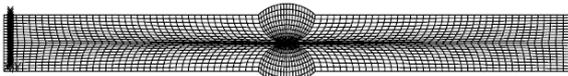


Figure 11. Double side butt weld (FRANC2D).

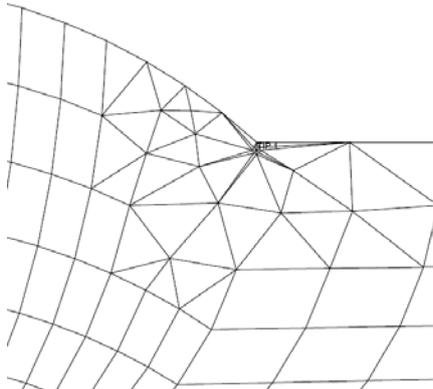


Figure 12. Screenshot from FRANC2D crack tip at weld toe,  $a_i = 0.1$  mm.

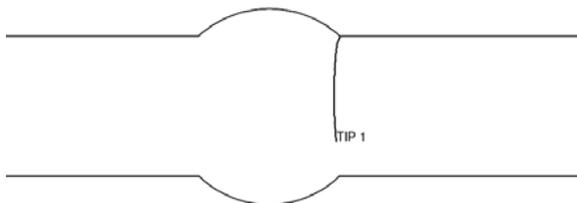
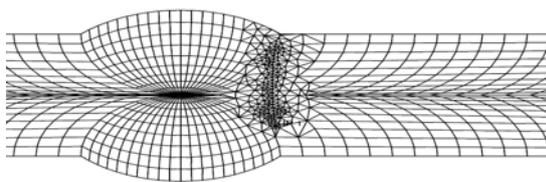


Figure 13. Screenshot from FRANC2D of the crack growth direction.



(a)



(b)

Figure 14. Screenshot from FRANC2D of the crack growth direction with mesh, a) Final crack propagation, b) Crack deformation and failure of butt welded joint.

6.3. Results of SIF Calculations

Cruciform joint and butt welded joint are the most conventional joints used in engineering structures. Accurate predictions of crack shape changes and fatigue lives require accurate SIF estimates because of the power-law nature of the Paris crack growth law. The determination of the SIF for the two dimensional cruciform and butt welded joints has been carried out and compared with empirical solutions using linear elastic finite element analysis.

6.3.1. Cruciform Joint

The results of the analytical study for the cruciform joints with equal main and attached plate thickness are plotted in Figure 15, based on the correction terms Eq. 5, and  $M_K$ , Eq. 6 together in Eq. 3. The results compared with the direct calculations of SIF as a function of crack length from FRANC2D. The results have been benchmarked as shown in Figure 16.

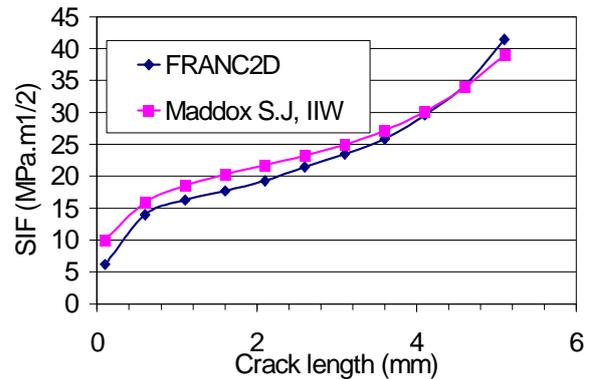


Figure 15. SIF as a function of crack length of cruciform joint fails from the weld toe compared with FEM.

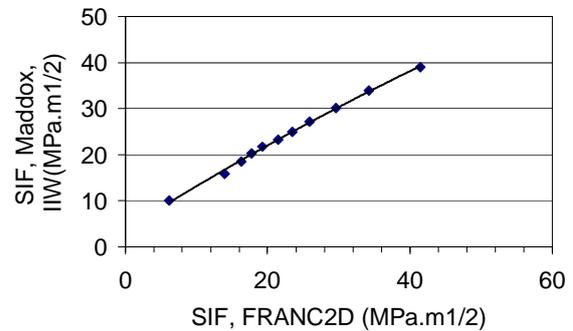


Figure 16. Comparison between analytical solutions and FEM.

6.4. Butt Weld

The comparison of results obtained for the butt weld is shown in Figure 17. It can be seen that the results from FEM (FRANC2D software) is close to the results obtained

from Eq. 9. FRANC2D appears to be more realistic and it compares well with empirical equations.

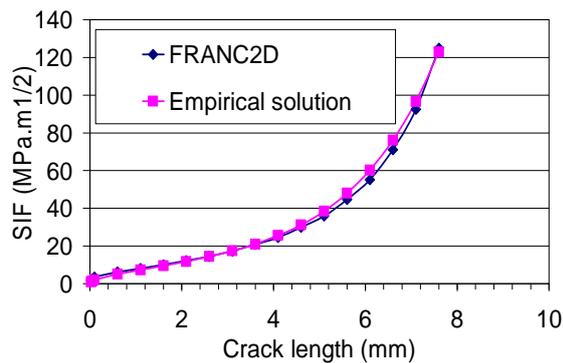


Figure 17. Comparison of FRANC2D and empirical equation of butt welded joint from ref [17].

## 7. Conclusions

The appropriate solutions for most welded joints are still difficult to find. Therefore SIF at of welded joints have been evaluated using the FEM.

The influence of the weld geometry was incorporated in the solution using FE analysis. The assumption was used that in the as-welded condition the crack remains open (mode-I) during the loading cycle due to the tensile residual stresses caused by welding are high enough. Therefore, the SIFs range corresponding to the nominal stress range is effective and independent of the  $R$ -ratio of nominal stresses. The FRANC2D software and quadrilateral elements were used to calculate the SIF of the joints from elementary of fracture mechanics.

This program has the ability to analyse the cracked body and describes the singularity a head of the crack tip. Thus, it can be concluded that for specific crack propagation, the SIF can be calculated under mode I-loading conditions. To demonstrate the efficiency of these calculations, two joints were investigated namely cruciform and butt weld joints. In the present study, an analytical approach for toe crack in cruciform joints has been developed.

The comparisons between this analytical approach and FEM have been agreed well.

Therefore, it can be advised to use the current FEM as combine with fracture mechanics to find appropriate SIFs solutions.

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