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Plane Deformation of a Textile Material with Boundary Forces Using Finite Element Method

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Abstract

An approach is proposed to determine the stress-strain state of soft shells having rectangular shape. The finite element method is applied to investigate the plane problem of strains and stresses calculation. The model is based on assuming that the density of strain potential energy of the textile shell material is a function of the macroscopic strain measures of the soft shell. The results obtained from the calculations display that the internal boundary layers begin at the vertices of the aperture. The calculations show that progressive cracks are developed due to the destruction of filaments forming the fabric along the boundary layers. The calculations are characterised by zero approximation and exact estimates of the maximum values of stresses are determined.

Keywords: Soft Shells; Stress; Strain; Finite Element; Zero Approximation; Boundary Layers.

1. Introduction

For the design and operation of textile industries [1, 2], studies are requested to investigate the stresses and strains resulted from the application of forces at the boundaries of a fabric having a rectangular form.

In order to calculate the actual strains of a fabric, a minimization of the energy functional are implemented which can be fulfilled by finite elements methods with a various choices of these elements [3, 4]. For each choice, this approach is based on the calculation of the nodal values of unknown functions as a solution of a system of non-linear algebraic equations [5, 6]. In this study, a reduction to a solution of Cauchy problem corresponding to some parameters which characterize the rigidity of the baric is used of the calculation of the actual strains. A zero value parameter relates to forces with negligible small elongations in the filaments of the fabric. By assuming a zero value of this parameter, the deformation of the fabric can be only resulting from the changing of angles between filaments. In the Cauchy problem approach, the initial conditions are taken as the deformations of the fabric at the zero value of the parameter. Different deformations of the fabric having a net structure with square meshes are obtained when such initial conditions are applied [7, 8]. By applying various boundary conditions, the fabric can be divided into zones with bi-axial or one-axial deflected mode, and these zones can be separated by concentration lines and disruption lines of the stresses. The elastic properties of the fabric are considered in the calculations which is the most essential condition to the zero order solution, which are observed in the surrounding of the concentration lines and disruption lines. To determine the geometric characteristics of the finite elements, the knowledge of the properties of these lines is of great importance.

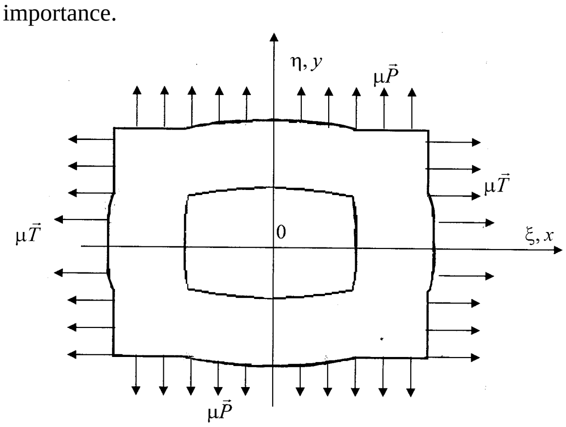

Figure 1. Deformed fabric with aperture.

In this study, a method is developed to investigate fabrics which have or haven't some inner cuts. Also, the conditions of the fabrics disruption at the cuts ends are studied. Further studies are essential to understand the local mechanical damages which produced from the manufacturing and the following processes, as these damages concentrate in areas of so small measure.

2. The Problem of Energy Functional Minimization

Let us consider the plane rectangular section of orthotropic shell including may be a rectangular aperture

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[9, 10]. The deflected mode of this section is supposed (see Fig.1) to be axially symmetric with respect to Ox -axis and Oy -axis, tensile loads $\mu\vec{T}$ and $\mu\vec{P}$ applied to the section edges are respectively parallel to the axes mentioned above, where μ is a parameter, which is introduced in such way that all investigation may be easily and exactly fulfilled if $\mu=0$.

For indication of plane location of any shell's particle two coordinate systems are used: Cartesian coordinate system Oxy and material (Lagrangian) system $O\xi\eta$ identifying shell's particles. We assume that material coordinates ξ,η of any shell's particle coincide with its Cartesian coordinates x, y when the shell is in non-deformed state.

In view of mentioned assumption it is possible to indicate the location of any shell's particle by equalities

$$x=\xi+u(\xi,\eta),\quad y=\eta+v(\xi,\eta), \tag{1}$$

where $u(\xi,\eta)$ and $v(\xi,\eta)$ are displacements of the shell's particle with Lagrangian coordinates ξ,η along Ox -axis and Oy -axis respectively.

Extensional strains of coordinate curves $\eta=const$ and $\xi=const$ are defined by expressions

$$\begin{aligned}\varepsilon_1&=\sqrt{\left(1+\frac{\partial u(\xi,\eta)}{\partial\xi}\right)^2+\left(\frac{\partial v(\xi,\eta)}{\partial\xi}\right)^2}-1,\\ \varepsilon_2&=\sqrt{\left(\frac{\partial u(\xi,\eta)}{\partial\eta}\right)^2+\left(1+\frac{\partial v(\xi,\eta)}{\partial\eta}\right)^2}-1.\end{aligned} \tag{2}$$

On the assumption of the absence of external forces distributed over the shell surface the following vector equilibrium equation takes place [1, 2]:

$$\frac{\partial}{\partial\xi}\left(\boldsymbol{\sigma}_1(\xi,\eta)\left|\frac{\partial\boldsymbol{r}(\xi,\eta)}{\partial\eta}\right|\right)+\frac{\partial}{\partial\eta}\left(\boldsymbol{\sigma}_2(\xi,\eta)\left|\frac{\partial\boldsymbol{r}(\xi,\eta)}{\partial\xi}\right|\right)=\boldsymbol{0} \tag{3}$$

Here

$$\boldsymbol{\sigma}_1(\xi,\eta)=\boldsymbol{e_1}\sigma_{11}+\boldsymbol{e_2}\sigma_{12},\quad \boldsymbol{\sigma}_2(\xi,\eta)=\boldsymbol{e_1}\sigma_{21}+\boldsymbol{e_2}\sigma_{22}, \tag{4}$$

Where

$$\boldsymbol{e_1}=\frac{\partial\boldsymbol{r}(\xi,\eta)}{\partial\xi}\Bigg/\left|\frac{\partial\boldsymbol{r}(\xi,\eta)}{\partial\xi}\right|,\quad \boldsymbol{e}_2=\frac{\partial\boldsymbol{r}(\xi,\eta)}{\partial\eta}\Bigg/\left|\frac{\partial\boldsymbol{r}(\xi,\eta)}{\partial\eta}\right|,$$

$\boldsymbol{r}(\xi,\eta)=x(\xi,\eta)\boldsymbol{i}+y(\xi,\eta)\boldsymbol{j}$ - is radius-vector (with respect to point O) of the shell's particle corresponding to Lagrangian coordinates ξ,η.

Let us consider the fabric section restricted by the lines $\xi=\mp L$ and the lines $\eta=\mp H$. Suppose that the rectangular aperture is enclosed between the lines $\xi=\mp l$ and the lines $\eta=\mp h$. It is quite reasonable to content ourselves with consideration of the quarter of the section located in the first quadrant of the plane Oxy and to set up the following zones. The first one is restricted by the lines $\xi=0$, $\xi=l$, $\eta=h$, $\eta=H$; the second one – by the lines $\xi=l$, $\xi=L$, $\eta=h$, $\eta=H$; the third one – by the lines $\xi=l$, $\xi=L$, $\eta=0$, $\eta=h$.

The analysis of the deflected mode of the soft shell is based on variational methods under the supposition of elasticity of the shell material; therefore the equilibrium state of the shell reduces its energy functional to a minimum. This functional can be represented in the form(equation 5):

$$U=\iint_S F\left(\mu,\frac{\partial u}{\partial\xi},\frac{\partial u}{\partial\eta},\frac{\partial v}{\partial\xi},\frac{\partial v}{\partial\eta}\right)d\xi d\eta-\mu T\int_0^H u(L,\eta)d\eta-\mu P\int_0^L v(\xi,H)d\xi$$

where the double integral is taken over the whole fabric, function F under the integral sign is the density of strain energy which is supposed to be known.

For greater definiteness of our considerations we shall assume, that the shell is linearly elastic network with rectangular cells. In this case it is possible to copy (5) in the form of (equation 6):

$$U=\iint_S\left(k_1\varepsilon_1^2+k_2\varepsilon_2^2\right)d\xi d\eta-\mu T\int_0^H u(L,\eta)d\eta-\mu P\int_0^L v(\xi,H)d\xi,$$

where k_1 and k_2 are the coefficients describing elastic properties of the fabric.

3. Finite Elements Method

Definitional domain of functions $u(\xi,\eta)$ and $v(\xi,\eta)$ is subjected to triangulation according to the scheme shown in Fig. 2.

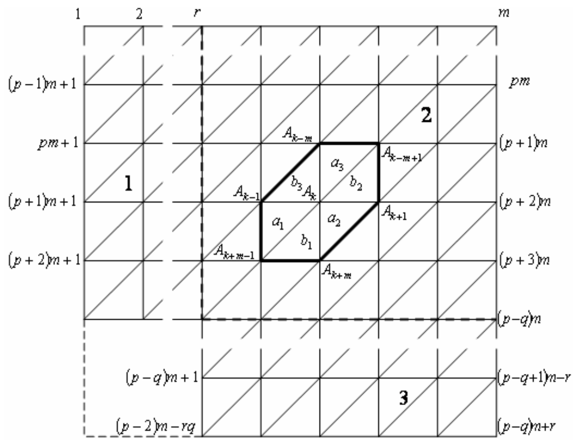


Figure 2. Triangulation of functions domain.

Without essential loss of generality one could suppose legs of all triangles to be equal, so that the following identities hold on [11, 12]:

$$s=\frac{L}{m}=\frac{l}{r}=\frac{H}{n}=\frac{h}{q} \tag{7}$$

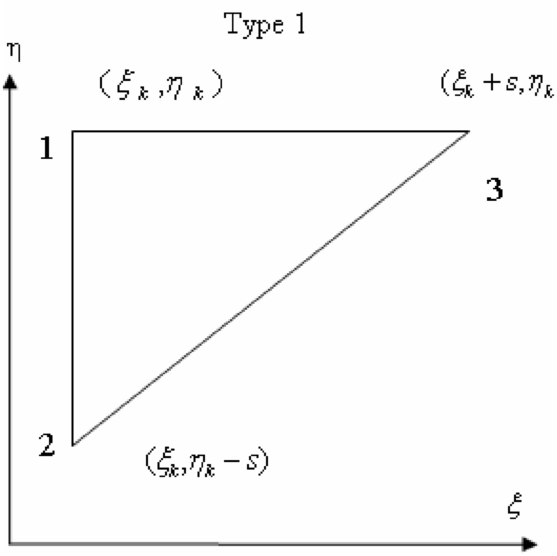
These equalities in particular define the sense of integers m, r, q and n.

As it can be seen in Fig.3, we deal with two types of elements. Let us introduce functions (equation 8):

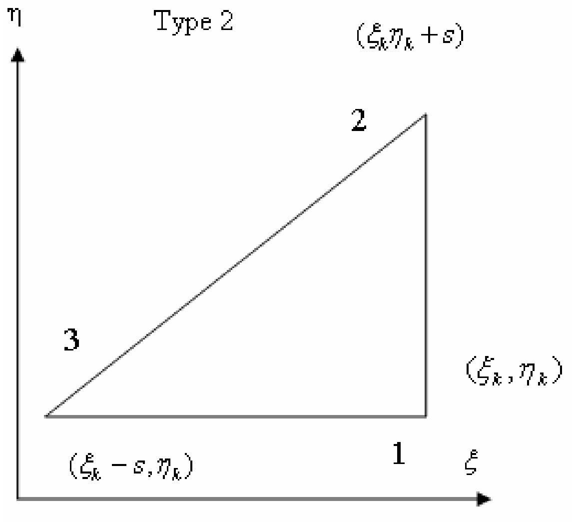
$$f_1(\xi,\eta;\xi_k,\eta_k)=1+\frac{1}{s}(\xi_k-\eta_k-\xi+\eta),$$

$$f_2(\xi,\eta;\xi_k,\eta_k)=\frac{1}{s}(\eta_k-\eta),\quad f_3(\xi,\eta;\xi_k,\eta_k)=\frac{1}{s}(\xi-\xi_k),$$

which distinctive feature is that f_i equals 1 in apex i and equals zero in other apexes, corresponding to the elements of type 1; everywhere outside of the element these functions are zero.



(a)



(b)

Figure 3. (a), (b). Two types of finite elements.

Similar functions correspond to the elements of type 2:

$$p_1(\xi,\eta,\xi_k,\eta_k)=1+\frac{1}{s}(\xi-\xi_k-\eta+\eta_k),$$
$$p_2(\xi,\eta,\xi_k,\eta_k)=\frac{1}{s}(\eta-\eta_k),\quad p_3(\xi,\eta,\xi_k,\eta_k)=\frac{1}{s}(\xi_k-\xi). \tag{9}$$

Now it is easy to obtain following approximation of displacements u and v:

$$u(\mu,\xi,\eta)=\sum_{k=1}^{MN-mn}u_k\breve{U}_k(\xi,\eta),\quad v(\mu,\xi,\eta)=\sum_{k=1}^{MN-mn}v_k\breve{V}_k(\xi,\eta), \tag{10}$$

where $\breve{U}_k$ and $\breve{V}_k$ are easily construed of functions given by (8) and (9).

Substitution of expressions (10) into energy functional (5) transmutes it into the function of required coefficients

$u_k = u_k(\mu) = u(\mu,\xi_k,\eta_k)$

and $v_k = v_k(\mu) = v(\mu,\xi_k,\eta_k)$.

By dint of equating the partial derivatives of this function with respect to mentioned coefficients u_k and v_k with zero, a system of algebraic equations could be obtained as the solution of which the coefficients could be found.

This system can be written as follows (equation 11):

$$\frac{\partial U}{\partial u_k}=2\iint_S\left(\frac{k_1\varepsilon_1}{1+\varepsilon_1}\left(1+\frac{\partial u}{\partial\xi}\right)\frac{\partial\breve{U}_k}{\partial\xi}+\frac{k_2\varepsilon_2}{1+\varepsilon_2}\frac{\partial u}{\partial\eta}\frac{\partial\breve{U}_k}{\partial\eta}\right)d\xi d\eta-\mu T\int_0^H\breve{U}_k(L,\eta)d\eta=0,$$
$$\frac{\partial U}{\partial v_k}=2\iint_S\left(\frac{k_1\varepsilon_1}{1+\varepsilon_1}\frac{\partial v}{\partial\xi}\frac{\partial\breve{V}_k}{\partial\xi}+\frac{k_2\varepsilon_2}{1+\varepsilon_2}\left(1+\frac{\partial v}{\partial\eta}\right)\frac{\partial\breve{V}_k}{\partial\eta}\right)d\xi d\eta-\mu P\int_0^H\breve{V}_k(\xi,H)d\xi=0.$$

4. Zero-Approximation Solution

In order to analyze this nonlinear system let us differentiate all equations with respect to parameter μ . As the result a system of linear equations in regard to derivatives $du_k / d\mu$ and $dv_k / d\mu$ is obtained.

This system is following:

$$C(\mu, X(\mu))\frac{dX}{d\mu}=B(\mu, X(\mu)), \tag{12}$$

where C is a matrix of a format $t\times t$, B is a vector of length t, X is a required vector of the same length $t=2((n-q)(m-1)+(m-r)(q-2))$.

Here the next designations are used

$$u_k = X_{2k-2},\quad v_k = X_{2k-1}, \tag{13}$$

while $k<2(m-1)(n-q)$.Otherwise

$$u_{m(n-q)+r}=X_{2[m(n-q)+r-1]},\quad v_{m(n-q)+r}=X_{2[m(n-q)+r]-1}. \tag{14}$$

Thus, the system (13) represents system of the ordinary differential equations from which should be found $u_k(\mu)$ and $v_k(\mu)$.

Solution of this system leads to Cauchy problem if $u_k(0)$ and $v_k(0)$, or more definitely $X_k(0)$, are known values.

Having written down (13) in the form of

$$\frac{dX}{d\mu}=C^{-1}(\mu, X(\mu))B(\mu, X(\mu)), \tag{15}$$

we can construct the decision under the recurrent formula (16):

$$X(\Delta\mu)=X(0)+\left.\frac{dX}{d\mu}\right|_{\mu=0}\Delta\mu=X(0)+C^{-1}(0,X(0))B(0,X(0))\Delta\mu,$$

$$X(\mu_k+\Delta\mu)=X(\mu_k)+\left.\frac{dX}{d\mu}\right|_{\mu=\mu_k}\Delta\mu=X(\mu_k)+C^{-1}(\mu_k,X(\mu_k))B(\mu_k,X(\mu_k))\Delta\mu.$$

Thus, calculation of the shell's deformation at any value μ can be carried out, if the decision of this problem is known at $\mu=0$, or in other words, at the first approximation.

Zero-approximation $X(0)$ required to begin calculations by formulas (16) is construed under supposition that to $\mu=0$ corresponds net-like fabric with square meshes made of non-extensible threads [13, 14]. In this case it is readily seen that for the first of above mentioned zones the following equations are valid:

$$u(\xi,\eta)=U_1(\xi),\quad v(\xi,\eta)=V_1(\xi) \tag{17}$$

For the third zone the equations

$$u(\xi,\eta)=U_2(\eta),\quad v(\xi,\eta)=V_2(\eta) \tag{18}$$

are satisfied.
For the second zone we have

$$u(\xi,\eta)=C_1,\quad v(\xi,\eta)=C_2 \tag{19}$$

where C_1 and C_2 are arbitrary constants subjected to search.

Let us write out the equilibrium equations for indicated zones. For the first zone one obtains that

$$\frac{\partial}{\partial\xi}\left(\sigma_{11}^{1}(\xi,\eta)\left(1+U_1'(\xi)\right)\right)=0 \tag{20}$$

$$\frac{\partial}{\partial\xi}\left(\sigma_{11}^{1}(\xi,\eta)V_1'(\xi)\right)+\frac{\partial}{\partial\eta}\left(\sigma_{22}^{1}(\xi,\eta)\right)=0 \tag{21}$$

The equilibrium equations for the third zone are given by

$$\frac{\partial}{\partial\xi}\left(\sigma_{11}^{3}(\xi,\eta)\right)+\frac{\partial}{\partial\eta}\left(\sigma_{22}^{3}(\xi,\eta)U_2'(\eta)\right)=0 \tag{22}$$

$$\frac{\partial}{\partial\eta}\left(\sigma_{22}^{3}(\xi,\eta)\left(1+V_2'(\eta)\right)\right)=0 \tag{23}$$

For the second zone the following equations are carried out

$$\frac{\partial}{\partial\xi}\left(\sigma_{11}^{2}(\xi,\eta)\right)=0\,,\quad \frac{\partial}{\partial\eta}\left(\sigma_{22}^{2}(\xi,\eta)\right)=0 \tag{24}$$

In equations (20) – (24) the upper index i, $i=1,2,3$, in designations σ_{11}^{i} and σ_{22}^{i} is used to identification of stresses in corresponding zone of the fabric section.
Equations (20)-(24) are easily integrated

$$\left.\begin{aligned}&\sigma_{11}^{1}(\xi,\eta)=\frac{s(\eta)}{1+U_1'(\xi)},\sigma_{22}^{1}(\xi,\eta)=-S(\eta)\frac{\partial}{\partial\xi}\left(\frac{V_1'(\xi)}{1+U_1'(\xi)}\right)+f(\xi),\\&S'(\eta)=s(\eta),\end{aligned}\right\} \tag{25}$$

$$\left.\begin{aligned}&\sigma_{22}^{3}(\xi,\eta)=\frac{q(\xi)}{1+V_2'(\eta)},\sigma_{11}^{3}(\xi,\eta)=-Q(\xi)\frac{\partial}{\partial\eta}\left(\frac{U_2'(\eta)}{1+V_2'(\eta)}\right)+g(\eta),\\&Q'(\xi)=q(\xi),\end{aligned}\right\} \tag{26}$$

$$\sigma_{11}^{2}(\xi,\eta)=G(\eta),\quad \sigma_{22}^{2}(\xi,\eta)=F(\xi) \tag{27}$$

where $f(\xi)$, $s(\eta)$, $q(\xi)$, $g(\eta)$, $F(\xi)$ and $G(\eta)$ are the desired functions.
According to the problem statement the following boundary conditions should be satisfied:

$$\sigma_{22}^{1}(\xi,h)=-S(h)\frac{\partial}{\partial\xi}\left(\frac{V_1'(\xi)}{1+U_1'(\xi)}\right)+f(\xi)\equiv 0 \tag{28}$$

$$\sigma_{22}^{1}(\xi,H)=-S(H)\frac{\partial}{\partial\xi}\left(\frac{V_1'(\xi)}{1+U_1'(\xi)}\right)+f(\xi)\equiv P \tag{29}$$

$$\sigma_{11}^{3}(l,\eta)=-Q(l)\frac{\partial}{\partial\eta}\left(\frac{U_2'(\eta)}{1+V_2'(\eta)}\right)+g(\eta)\equiv 0 \tag{30}$$

$$\sigma_{11}^{3}(L,\eta)=-Q(L)\frac{\partial}{\partial\eta}\left(\frac{U_2'(\eta)}{1+V_2'(\eta)}\right)+g(\eta)\equiv T \tag{31}$$

$$\sigma_{11}^{2}(L,\eta)=G(\eta)\equiv T,\quad \sigma_{22}^{2}(\xi,H)=F(\xi)\equiv P \tag{32}$$

Furthermore, on the border of the first and the second zone as well as on the border of the second and the third zone the conditions of normal stresses conjunction should be satisfied. Satisfying these and also geometrical conditions of zones conjunction, as well as the symmetry conditions one can easily obtain that

$$\left.\begin{aligned}&U_1(\xi)=-\xi+\frac{T(H-h)}{P}\ln\left(\frac{P}{T(H-h)}\xi+\sqrt{1+\left(\frac{P}{T(H-h)}\xi\right)^2}\right),\\&C_1=U_1(l)\end{aligned}\right\} \tag{33}$$

$$\left.\begin{aligned}&V_2(\eta)=-\eta+\frac{P(L-l)}{T}\ln\left(\frac{T}{P(L-l)}\eta+\sqrt{1+\left(\frac{T}{P(L-l)}\eta\right)^2}\right),\\&C_2=V_2(h)\end{aligned}\right\} \tag{34}$$

$$\begin{aligned}&\sigma_{11}^{1}(\xi,\eta)=T\sqrt{1+\left(\frac{P}{T(H-h)}\xi\right)^2}\,,\\&\sigma_{22}^{3}(\xi,\eta)=P\sqrt{1+\left(\frac{T}{P(L-l)}\eta\right)^2}\end{aligned} \tag{35}$$

The displacements of the shell's particles $V_1(\xi)$, $U_2(\eta)$ should be obtained by dint of equations (25), (26) which accomplishes exact investigation of zero-approximation. This approximation is used as initial condition when the desired solution is numerically construed by continuation by parameter μ.

Obtained results allow us to calculate in zero-approximation coefficients $u_k(0)$ and $v_k(0)$.
For considered textile structure of fabric the stresses are defined by formulas:

$$\boldsymbol{\sigma}_1=\mathbf{e}_1\sigma_{11}=2k_1\varepsilon_1\mathbf{e}_1,\quad \boldsymbol{\sigma}_2=\mathbf{e}_2\sigma_{22}=2k_2\varepsilon_2\mathbf{e}_2,\quad \sigma_{12}=\sigma_{21}=0. \tag{36}$$

The calculations performed in this paper have shown, that the internal boundary layers beginning at vertices of the aperture and dividing specified above zones arose.

Along these lines destruction of filaments forming the fabric leads to development of progressive cracks. In zero approximation exact estimations of the maximal values of stresses are received.

5. Conclusions

The strains of the fabric were calculated using finite element methods. In the calculations the elastic properties of the fabric are considered. The model developed takes into account the types of fabrics which posses some inner cuts also the model applies for fabrics haven't such inner cuts. Referring to the different boundary conditions, the fabric is divided into stress deformed states which are bi-axial or uni-axial. Lines of concentrations as well as lines of disruption are used to describe the boundaries of the stress deformed zones. The conditions expected from the calculations based on the zero ordered solution are found to be applicable in the vicinity of the lines of concentration and lines of disruption. The calculations showed also that the conditions of fabric disruption appease at the end of the cuts.

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