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# Mathematical Modeling for Pump Controlled System of Hydraulic Drive Unit of Single Bucket Excavator Digging Mechanism

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#### Abstract

Industrial robots have turned out to be an everyday occurrence for engineers during the last twenty years. Increasing efficiency is one of the hottest research topics in hydraulic system. The comparison between two types of hydraulic control systems, which are the valve, controlled system and the pump-controlled system is essential area of researching. Hydraulic actuator with pump-controlled system is equipped in high power mining excavators and forestry equipment in order to increase efficiency. The purpose of this paper is to build a mathematical model for a hydraulic actuator with pump-controlled system. The boom of single bucket excavator is modeled and simulated as an example. The scheme of this system is presented and itemized. The model of these items is simulated and the control circuits are presented as well as the simulation results.

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## 1. Introduction

Hydraulic actuation devices may be linear or rotary and are usually referred to as pistons or motors, respectively. A pump or a valve giving four basic hydraulic power elements and two basic over-all systems may control these two actuation devices: pump controlled and valve controlled. Such hydraulic power elements is simply a combination the of principal power device in all hydraulic systems [1].

The pump-controlled system consists of a variable delivery pump supplying fluid to an actuation device. The fluid flow is controlled by the stroke of the pump to vary output speed and the pressure generated matches the load. It is usually difficult to closely couple the pump to the actuator and this causes large contained volumes and slow response [2].

The valve-controlled system consists of a servo valve controlling the flow from a hydraulic power supply to an actuation device. The hydraulic power supply is usually a constant pressure type (as opposed to constant flow) and there are two basic configurations. One consists of a constant delivery pump with a relief valve to regulate pressure whereas the other is much more efficient because

it uses a variable delivery pump with a stroke control to regulate pressure.

The features of each system tend to complement the other so that application requirements would dictate the choice to be made. Generally, there is not a cost advantage to either because the need for a replenishing arrangement and a stroke servo for the pump controlled system offset the costly servo valve and heat exchangers required for the valve controlled system. However the faster response capability of valve controlled systems both to valve and load inputs makes this arrangement preferred in the majority of applications in spite of its lower theoretical maximum operation efficiency of 67% in low power applications where the inefficiency is comparatively less important, use of valve controlled system is nearly universal. Applications, which require large horsepower for control purposes usually, do not require fast response so that a pump-controlled system is preferred because of its superior theoretical maximum operating efficiency of 100% [3].

The pump-controlled actuator represents a further alternative. A simplified structure is depicted in Figure 1. This actuator uses a servo pump as a final control element in the actuator closed loop control. Similar to the classical circuit of a hydrostatic transmission, a closed hydraulic circuit design is employed for the realization of a four-

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quadrant operation. This makes the utilization of brake energy in case of aiding loads generally possible. The pump controller adjusts the pump displacement according to the demanded volumetric flow, which depends on the actuator velocity. The pressure difference between the high pressure and the low-pressure line is automatically given by the actuator load. Mostly, the low pressure is set constant with the help of an external pump, which can also be employed for the supply of the pump control system. Thereby the hydraulic output power of the pump is simultaneously adapted to the required mechanical output power of the actuator. Due to its simple design characterized by small number of parts, and excellent achievable dynamic performance, the swash plate of the axial piston pump would be usually the best choice for the servo pump [4].

The application of the displacement control principle for actuators of heavy-duty mobile manipulators has the following three important advantages:

- The improved utilization of primary energy due to the omitted valves,
- The possibility of the additional use of brake energy in case of aiding loads,
- The reduction of total system weight due to the replacement of one or two large pumps by a number of smaller pumps [5].

The realization of pump control is uncomplicated for rotary actuators and linear actuators with double rod cylinders. Several industrial applications have been developed recently. The kinematics and the whole machine design of mobile manipulators require very often the use of differential cylinders. The unequal areas of the differential cylinder have to be compensated, if the differential cylinder may run within a pump-controlled actuator in a closed hydraulic circuit as shown in Figure 1B. The closed hydraulic circuit permits the four-quadrant operation of the pump-controlled actuator. Different circuit solutions allowing the compensation of the unequal cylinder areas were developed recently. The first solution shown in Figure 1A was developed by Lodewyks at the University of Aachen in 1993. A hydraulic transformer (5) serves as a flow compensator. The servo pump (2) has been used as a final control element in the closed loop position control of the linear actuator. The second solution is shown in Figure 1B characterized by two variable displacement pump units (1, 2) for one cylinder. This concept requires a multi-variable control concept for the simultaneous pressure and position control of the actuator to guarantee the required flow compensation.

A third solution showed that in Figure 1C represents an enlargement of the solution with two servo pumps. The circuit is supplemented by a third variable displacement pump (3) and a proportional pressure valve allowing the adjustment of the pressure level. This solution requires a high number of components but does not need a multivariable control concept. These actuator solutions were developed for special industrial applications. For the majority of applications in mobile machines, the replacement of a valve controlled actuator by a displacement-controlled actuator, which requires two and more variable displacement pumps and perhaps even further elements, which increase the cost [4].

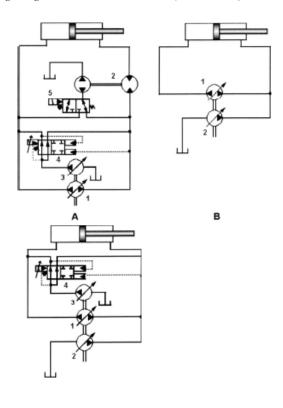


Figure 1. Circuit solutions for pump controlled actuator with differential cylinder

The purpose of this paper is to build a mathematical model for a hydraulic actuator with pump-controlled system. The scheme of this system is presented and itemized. Models of items and simulation of control circuits are also presented as well as the result of the simulation

# 2. Hydraulic Drive Unit of Single Bucket Excavator Digging Mechanism

Kinematics and hydraulic circuit diagram of the hydraulic excavator digging mechanism is shown in Figure 2 and Figure 3. The hydraulic excavator digging mechanism consists of four links, boom 'S', connecting element 'V', stick 'R', and bucket 'K'; each is set in motion by its own hydraulic cylinder. A variable displacement pump controls the velocity of the piston rod of the cylinder.

After investigating the load of each cylinder, it was shown that the boom cylinder is carrying the highest load [1]. Each element has its own mass, which in turn has its own reaction on the boom

cylinder. Kinematics of digging mechanism and structure design show that the reaction of the mass of each digging element changes with the stroke of boom cylinder. As shown in Figure 2, it is clear that any change in the mechanism element position will result in substantial changes in the position of the mass center consequently this will cause changes in inertia and the equivalent mass as well as the total contained oil volume.

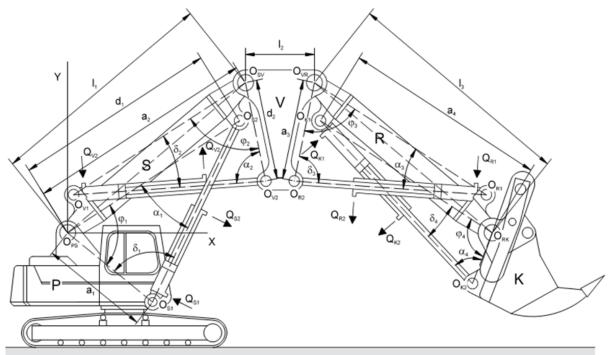


Figure 2 Excavator Digging Mechanism, (Boom S, Connecting element V, Stick R, and Bucket K) [1].

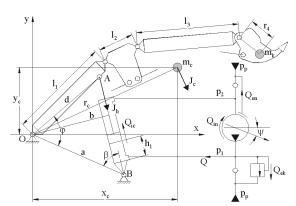


Figure 3. Hydraulic Unit and cinematic circuit diagram of the boom mechanism

Thus, this system is characterized by variable parameters depending on the elements position. The control system with dynamic characteristics requires designing a control algorithm structure (dynamic controller), which must be efficient to compensate the dynamic variation of the controlled object parameters.

The following assumptions were made during this paper:

The friction force at the hinges and the seals are neglected. The link of each mechanism is considered as a rigid body. The return pressure is constant and equal the replenishing pressure  $P_2$ =0.

The parameters changing range of the boom mechanism can be computed from its mathematical model. The mathematical model of the boom hydraulic cylinder is set up for forward stroke movement since digging process and consequently the highest load occurs during forward stroke.

# 3. Mathematical Modeling of the Hydraulic Drive Unit with Pump Controlled System

Mathematical model describes in details the boom hydro cylinder forward stroke and displays all-important characteristics of the investigated object. The schematic diagram of the model is shown in Figure 4.

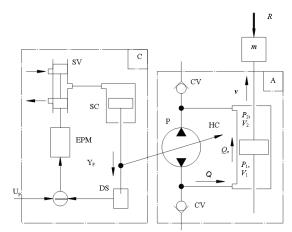


Figure 4. Model of the hydraulic actuator with pump controlled system

The model incorporates:

- a) Control object cylinder HC and speed sensor SS to measure the rod speed. The cylinder is loaded by active force R (h) and mass m (h), since these parameters are a function of the cylinder stroke position h.
- Power supply unit, which is represented by the variable displacement, pump P.

Servo valve unit, which converts input signal 'U<sub>p</sub>' and feedback signal from sensor displacement sensor 'DS' into a control signal 'Y<sub>p</sub>'.

### 3.1. Model Description

The model consists of Electro hydraulic Servo valve 'C' to control the flow of variable displacement pump 'P'. An input signal 'U<sub>p</sub>' is activated by the excavator operator which enters the amplifier 'EPM', and then the servo valve spool starts to move. Oil enters into the servo valve hydro cylinder piston chamber 'SC' forcing the rod, which is linked to the pump swash plate forward and thus changing the pump flow. The rod displacement 'Y<sub>p</sub>' will keep changing until the electric signal of displacement sensor 'DS' balances the control signal 'U<sub>p</sub>'. Oil flow from the pump 'P' enters into the cylinder chamber, creating force enough to move the mass against external force 'R'. Check valves 'CV' prevent stagnation, compensate the shortage of the flow from the replenishing flow, and allow the excess flow to pass through the pressure relief valve to the tank

#### 3.2. Mathematical Modeling of the Actuator

Equation of movement

$$m(h)\frac{dv}{dt} = A \cdot P_1 - R \tag{1}$$

Continuity equation of the oil in the cylinders chambers

$$\frac{V_1}{E} \frac{dp_1}{dt} = k \cdot y_p - C_e \cdot P_1 - A \cdot v \tag{2}$$

Servo valve equation

$$T_{p} \frac{dy_{p}}{dt} + y_{p} = K_{p} \cdot U_{p}$$
(3)

Where: m(h) is the mass as a function of the digging element position (h), v is the velocity of piston rod measured by speed sensor SS, A area of the piston, R external force,  $V_1$  volume of hydro cylinder chamber, E oil bulk modulus, k pump coefficient (which can be computed experimentally from the relationship of the pump flow Q and the servo valve displacement  $Y_p$ ),  $K_p$  controller coefficient (which can be computed experimentally from the relationship of the servo valve displacement  $Y_p$  and input signal  $U_p$ ), t time.

Combining equations 1, 2, and 3, the mathematical model will have the following form:

$$m_{eq}(h)\frac{dv}{dt} = A_1P_1 - A_2P_2 - R(h)$$
 (4)

$$\frac{V(h)}{E}\frac{dP_1}{dt} = k\psi - (C_e + C_i)(P_1 - P_2) - A_1v,$$
 (5)

Where:  $m_{eq}$  is the equivalent or the effective mass of the digging mechanism elements acting on the cylinder piston rod, kg.

Equivalent mass acting on hydro cylinder rod changes depending on digging mechanism element positions as shown in equation a below.

It is assumed that all digging mechanism elements are joint rigidly as one unit and rotating as solid body with mass  $m_c$  around axis through O.  $m_c = m_1 + m_2 + m_3 + m_4$ .  $m_{eq}(h)$  can be computed from the kinematics and dynamics analysis of the system given in [1]. h is the vector of digging mechanism state position characterized by cylinder stroke,  $h_1$  boom,  $h_2$  insert,  $h_3$  stick and  $h_4$  bucket.

In addition, from [2] the equivalent mass will have the following relationship:

$$m_{eq}(\vec{h})b\frac{d^2h_1}{dt^2} = m_c r_c(\vec{h})w_{\tau}$$
, (a)

Kinematics of Fig. 3 gives

$$b = \frac{a d \sin \varphi}{\sqrt{a^2 + d^2 - 2a d \cos \varphi}},$$
 (b)

$$\varphi = \arccos \frac{a^2 + d^2 - (h_{01} + h_1)^2}{2ad}$$
 (c)

Then the angular acceleration can be computed as:

$$w_{\tau} = r_{c} \frac{d^{2} \varphi}{dt^{2}} = \frac{2r_{c}(h_{01} + h_{1})}{\sqrt{4a^{2}d^{2} - [a^{2} + d^{2} - (h_{01} + h_{1})^{2}]^{2}}} \frac{d^{2}h_{1}}{dt^{2}} \qquad (e)$$

From equation (b), (c) and (e), it is possible to compute the maximum and minimum limits of equation (a) which can be rewritten as:

$$m_{eq} = \frac{a^2 + d^2 - 2a \ d \ \cos\varphi}{a^2 d^2 \sin^2\varphi} \cdot r_C^2 m_C.$$
 (d)

#### Where:

- r<sub>c</sub>, is the radius of rotation around hinge O,
- $w_{\tau}$  is the angular acceleration of the center of mass,
- m<sub>c</sub> is the mass of the system.
- A<sub>1</sub>, A<sub>2</sub> are the areas of forward and return stroke of the piston, m<sup>2</sup>.
- P<sub>1</sub>, P<sub>2</sub> are the pressure at the piston and rod chambers, Pa.
- P<sub>2</sub> equals the replenishing pressure P<sub>2</sub>=0 (assumed).
- k is the pump control element (swash plate) gain coefficient m³/rad/deg.
- v is the piston rod velocity, m/s.
- V is the total contained volume including the chambers and the connecting hoses and manifolds.
- E is the liquid bulk modulus of elasticity (constant).
- $\psi$  is the swash plate parameter  $(0 \le \psi \le 1)$ .
- C<sub>e</sub>, C<sub>i</sub> is the lumped external and internal leakage coefficients respectively.
- h<sub>01</sub> distance from cylinder hinge B to the piston return stroke position.

Equations 4 and 5 can be Lap laced and written in the general form as:

$$\left(\frac{m_{eq}\left(\overline{h}\right)V(h_1)}{EA_1^2}s^2 + \frac{m_{eq}\left(\overline{h}\right)(C_e + C_i)}{A_1^2}s + 1\right) \cdot \delta v = k \cdot \delta \psi \qquad (6)$$

Where:  $\delta$  deviation from steady state value, s Lap lace operator.

The dynamic behavior of this second order system can be then described in terms of two parameters, T which is the characteristic time, and  $\xi$  which is the damping ratio. The system characteristic equation can be rewritten to give:

$$\frac{\mathbf{m}_{eq}(\mathbf{h})\mathbf{V}(\mathbf{h}_1)}{\mathbf{E}\mathbf{A}_1^2} = \mathbf{T}^2 \tag{7}$$

$$\frac{m_{eq}(h)(C_e + C_i)}{A_1^2} = 2T\xi$$
 (8)

## 4. Simulation Example.

The given analysis allows computing the important parameters of the controlled object at any digging mechanism position resulting from changing vector  $\boldsymbol{h}$ .

All the data are collected from real mining excavator. The calculations were performed for the following real dimensions collected from 15-m3 single bucket excavator in [1].

Pump leakage coefficients were evaluated based on lab experiment from [1] as follow:

$$C_{ep}=2.24\cdot10^{-11}m^5/(N\cdot s); C_{ip}=5.22\cdot10^{-11}m^5/(N\cdot s)$$

The following coefficients were collected from manufacturers catalogues for the boom cylinder and the pressure relief valve.

Relief valve external leakage coefficient  $C_{er}$ =0.026·10<sup>-11</sup>m<sup>5</sup>/(N·s), no internal leakage coefficient was considered. Hydraulic cylinder internal leakage coefficient  $C_{ic}$ =0.137·10<sup>-11</sup>m<sup>5</sup>/(N.s), no external leakage coefficient was considered.

The lumped leakage coefficients are:

Total external leakage coefficient  $C_e = C_{ep} + C_{er}$ ; and Total internal leakage coefficient  $C_i = C_{ip} + C_{ic}$ . For MEXZ 4AP112M-5 mining excavator shown in Fig.1.

Simulation of the system of equations 6,7 and 8 for T and  $\xi$  were accomplished with step  $\Delta h_1$ =0.1H for the boom digging mechanism element (H is the maximum stroke of the boom cylinder).

In Figure 5 the result of simulation of the pump controlled system which have been made for the given data from real mining excavator, the pump Hytos MLPD/G217D, the proportional valve Bosch NG6, the linear hydraulic actuator Mannesmann Rexroth CYW 160 B 63/45-120, volume of the pipe 0.002 m<sup>3</sup> and the dynamical load described above.

Maximum and minimum values of T and their corresponding  $\xi$  ,  $m_{eq}$  , V, were obtained as follows:

 $T_{\text{max}} = 0.102$ ;  $\xi = 0.015$ ;  $m_{\text{eq}} = 1.25 \times 10^6$ ; V = 0.628;

 $T_{min}$ =0.0066;  $\xi$ =0.012;  $m_{eq}$ =0.066·10<sup>6</sup>; V=0.050;

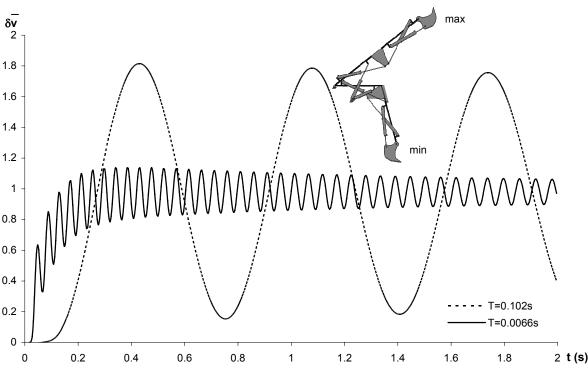


Figure (5) Transient response for hydraulic drive

#### 5. Discussion And Conclusion

The transient response for two extreme boom positions in compliance with actuator mathematical model was resolved and analyzed to give the following results:

The digging mechanism load acting on the boom cylinder rod was changed from 0.32\*10<sup>6</sup>N to 2.38\*10<sup>6</sup>N.

The value of the characteristic time of the linearized model of the control object was changed from  $T_{min} = 6.6*10^{-3} s$  to  $T_{max} = 102*10^{-3} s$ .

The damping coefficient was also changed from  $\xi_{min}{=}0.012$  to  $\xi_{max}{=}0.015.$ 

The dynamic behavior of the load, the characteristic time, and the damping coefficient would greatly affect the transient process parameters thus, for each piston movement, a new transient process with different parameters would appear.

The digging mechanism and hydraulic unit mathematical model represents a dynamic system with variable parameters because the coefficients are determined by the digging mechanism position.

The simulation of the hydraulic actuator with pump controlled model shown in Fig.5 were obtained without correction for two extreme digging mechanism positions and it was clearly shown the weak damping of hydraulic unit resulted in sustained vibration.

The simulation of the model is employed to show that hydraulic actuator with open loop control system is not relevant for hydraulic unit with pump controlled system. It is necessary to construct a rational dynamic regulator to control the pump flow in order to stabilize the system.

The next step is to look for a suitable controller for the hydraulic drive digging mechanism to eliminate the vibration resulting from the weak damping.

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